2導体分布結合線路とLC素子から成る回路の構成

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Synthesis of Networks Consisting of LC Elements and Coupled 2-Wire Lines

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Necessary and sufficient conditions are presented under which a 2×2 2-variable admittance matrix may be realized as a 2-port network in which two distinct 4-port networks consisting of LC elements only are terminated in a 2-port network consisting of coupled 2-wire lines. Furthermore, an example is given to illustrate the synthesis technique.

Key words: Circuit theory, Lumpled and distributed mixed networks, Multivariable positive real functions and matrices.

1 Introduction

It is well known that networks containing both lumped and distributed elements may be analyzed and synthesized by using the theory of multivariable positive-real functions or matrices.[1-4] A very important practical class of these networks, especially at microwave frequencies, is a cascaded structure of transmission lines, separated by lumped 2-port networks. Realizability conditions on various cascaded structures were presented by several authors. [3-8], [10-12]

Recently, H.Fujimoto has considered three distinct cases of lumped and distributed mixed 2-port networks, and has presented those realizability conditions.[13,14] Each of the networks consists of a cascade-load connection that a distributed constant 2-port network, as in the following three cases, terminates a lumped constant 4-port network consisting of five capacitors. The cases are with respect to both wires of the coupled line at the far end : one is ground-connected, the second is open-circuited and the third is short-circuited.

In this paper, realizability conditions are derived under which a 2-variable admittance matrix of complex variables "p" and "s" may be realized as a 2-variable 2-port network in which an s-variable 4-port network is terminated in a p-variable 2-port network(Fig.1). The s-variable 4-port network, which is referred to as fifth-order elliptic low-pass filters, is constructed of LC-elements. The p-variable 2-port network consists of a cascade of two coupled 2-wire lines, and both wires of the second coupled line are ground-connected at the far end. Furthermore an example is given to illustrate the synthesis technique.

2 Circuit Analysis

In this section, relationships with respect to the admittance matrix of the 2-port network shown in Fig.1 are derived. Fig.2 shows the 2-port network which is a configuration of a 4-port network $\overline{\mathbb{N}}$ terminated in a 2-port network $\widetilde{\mathbb{N}}$. The 2-port network will be described as $\mathbb{N}$. We assume $\overline{\mathbb{N}}$ and $\widetilde{\mathbb{N}}$ are linear, passive, time-invariant, lossless and reciprocal.

Let $\mathbb{W} = [w_{ij}]$ be a 2×2 admittance matrix of $\mathbb{N}$ between ports 1 and 2 in Fig.2. Furthermore, let $\overline{\mathbb{Y}} = [\overline{y}_{ij}]$ and $\widetilde{\mathbb{Y}} = [\widetilde{y}_{ij}]$ be 4×4 and 2×2 admittance matrices of $\overline{\mathbb{N}}$ and $\widetilde{\mathbb{N}}$, respectively.

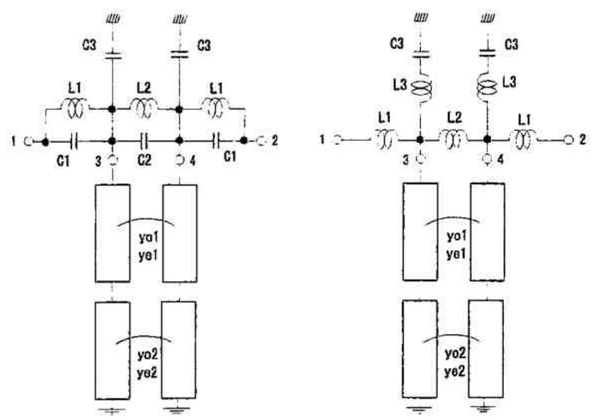

Fig.1 Lumped and distributed mixed 2-port networks.

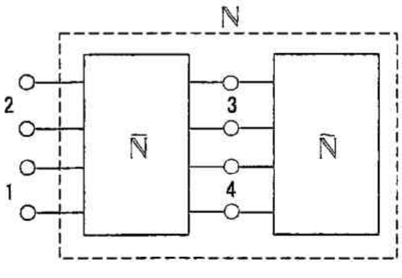

Fig.2 2-port netwok $\mathbb{N}$ where $\widetilde{\mathbb{N}}$ terminates $\overline{\mathbb{N}}$.

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After some matrix manipulation, the matrix $\mathbb{W}$ may be expressed in the form

$$\mathbb{W}=\overline{\mathbb{Y}}_{11}-\overline{\mathbb{Y}}_{12}\left(\overline{\mathbb{Y}}_{22}+\widetilde{\mathbb{Y}}\right)^{-1}\overline{\mathbb{Y}}_{21}$$

where

$$\overline{\mathbb{Y}}_{11}=\begin{bmatrix}\bar{y}_{11} & \bar{y}_{12}\\ \bar{y}_{12} & \bar{y}_{22}\end{bmatrix},\ \overline{\mathbb{Y}}_{22}=\begin{bmatrix}\bar{y}_{33} & \bar{y}_{34}\\ \bar{y}_{34} & \bar{y}_{44}\end{bmatrix}$$

and

$$\overline{\mathbb{Y}}_{12}=\overline{\mathbb{Y}}_{21}^{T}=\begin{bmatrix}\bar{y}_{13} & \bar{y}_{14}\\ \bar{y}_{23} & \bar{y}_{24}\end{bmatrix}.$$

The superscript "T" denotes the transpose of any matrix. Assume $\bar{y}_{13}=\bar{y}_{24}$ and $\bar{y}_{14}=\bar{y}_{23}$, then we can have the following with respect to elements of $\mathbb{W}$.

$$w_{11}\pm w_{12}$$
$$=(\bar{y}_{11}\pm\bar{y}_{12})-\frac{(\bar{y}_{13}\pm\bar{y}_{14})^2}{(\bar{y}_{33}\pm\bar{y}_{34})+(\widetilde{y}_{11}\pm\widetilde{y}_{12})} \tag{1}$$

where only the upper or only the lower signs are to be used.

3 Realizability Conditions

Necessary and sufficient conditions are presented for the realizability of 2-port networks shown in Fig.1 in the standpoint of functions of two complex variables "p" and "s". Symbols "p" and "s" are used to express Richards' transformation $\tanh \tau s$ ($\tau>0$) and the complex frequency, respectively.

The following theorem, which is the extension of Richards and Saito's theorem, will be applied in order to extract a coupled 2-wire line from a multivariable positive-real matrix given.

Theorem 1 [8,9] **:** Let $\mathbb{W}(p,\widehat{q})$ be an $r\times r$ symmetric positive-real matrix in complex variables p, $\widehat{q}=(q_1,\cdots,q_m)$. If there exists a matrix $\mathbb{R}_0$ and a positive constant $p=p_0$ such that

$$\mathbb{W}(p_0,\widehat{q})=\mathbb{R}_0$$

then $\mathbb{W}_{\mathbf{R}}(p,\widehat{q})$ which is defined by

$$\mathbb{W}_{\mathbf{R}}(p,\widehat{q})=$$
$$\mathbb{R}_0\left[\mathbb{R}_0-p\mathbb{W}(p,\widehat{q})\right]^{-1}\left[\mathbb{W}(p,\widehat{q})-p\mathbb{R}_0\right]$$

is also an $r\times r$ positive-real matrix if the inverse exists, and in general,

$$deg_p\mathbb{W}_{\mathbf{R}}(p,\widehat{q})<deg_p\mathbb{W}(p,\widehat{q}).$$

In particular if

$$\mathbb{W}(-p_0,\widehat{q})=-\mathbb{R}_0$$

then

$$deg_p\mathbb{W}_{\mathbf{R}}(p,\widehat{q})=deg_p\mathbb{W}(p,\widehat{q})-1.$$

We will present conditions for realizing network configurations shown in Fig.1.

Theorem 2: Necessary and sufficient conditions for a reactance 2×2 symmetric matrix $\mathbb{W}(p,s)=[w_{ij}(p,s)]$ $(i,j=1,2)$ to be realized as the admittance matrix of the 2-port network between ports 1 and 2 in Fig.1(a) are that
(1) $w_{11}(p,s)=w_{22}(p,s)$
(2) for $k=1,2$,

$$\frac{1}{w_{11}(p,s)\pm w_{12}(p,s)}$$
$$=\frac{1}{\alpha_k s+\dfrac{\beta_k}{s}}+\frac{1}{\gamma_k\, s+\dfrac{\delta_k}{s}+a_k p+\dfrac{b_k}{p}} \tag{2}$$

where (i) $\alpha_1=\alpha_2>0,\ \beta_1=\beta_2>0,\ \gamma_2>\gamma_1>0,$ $\delta_1=0,\ \delta_2>0$, (ii) $a_1+b_1>a_2+b_2>0$, $a_2b_1(a_1+b_1)>a_1b_2(a_2+b_2)>0$ for $a_k,b_k>0$. When "+", $k=1$ must be adapted, and when "−", $k=2$.

Proof: The proof of necessity will be omitted here. Therefore, the proof is presented only for the sufficient conditions, and is performed through three stages: the first is to yield elements of the admittance matrix in eqn.(1), the second is to obtain values of LC elements, and the last is to provide characteristic admittances of coupled 2-wire lines.

First of all to obtain elements of the admittance matrix of eqn.(1), we rewrite eqn.(2) as in the following:

$$w_{11}(p,s)\pm w_{12}(p,s)=$$
$$\alpha_k s+\frac{\beta_k}{s}-\frac{\left(\alpha_k s+\dfrac{\beta_k}{s}\right)^2}{(\alpha_k+\gamma_k)\,s+\dfrac{\beta_k+\delta_k}{s}+a_k p+\dfrac{b_k}{p}}. \tag{3}$$

Comparing eqn.(3) with eqn.(1) yields the following relationships:

$$\bar{y}_{11}\pm\bar{y}_{12}=\alpha_k s+\frac{\beta_k}{s} \tag{4}$$

$$\bar{y}_{13}\pm\bar{y}_{14}=\varepsilon\left(\alpha_k s+\frac{\beta_k}{s}\right)\quad(\varepsilon=\pm1) \tag{5}$$

$$\bar{y}_{33}\pm\bar{y}_{34}=(\alpha_k+\gamma_k)\,s+\frac{\beta_k+\delta_k}{s} \tag{6}$$

$$\widetilde{y}_{11} \pm \widetilde{y}_{12} = a_k p + \frac{b_k}{p}. \qquad (7)$$

From eqn.(4) we have

$$\bar{y}_{11} = \alpha_1 s + \frac{\beta_1}{s}$$

and

$$\bar{y}_{12} \equiv 0.$$

To obtain $\bar{y}_{13}$ and $\bar{y}_{14}$ from eqn.(5), we will adopt $\varepsilon = -1$. Then,

$$\bar{y}_{13} = -\left(\alpha_1 s + \frac{\beta_1}{s}\right)$$

and

$$\bar{y}_{14} \equiv 0.$$

From eqn.(6),

$$\bar{y}_{33} = \left(\alpha_1 + \frac{\gamma_1 + \gamma_2}{2}\right) s + \left(\beta_1 + \frac{\delta_2}{2}\right) \frac{1}{s}$$

and

$$\bar{y}_{34} = -\frac{\gamma_2 - \gamma_1}{2} s - \frac{\delta_2}{2} \frac{1}{s}.$$

To obtain LC element values of the second stage, we construct the 4×4 admittance matrix $\overline{\mathbb{Y}}(s)$ of $\overline{\mathbb{N}}$ in Fig.2. The result is

$$\overline{\mathbb{Y}}(s) = \overline{\mathbb{H}}_C s + \frac{\overline{\mathbb{H}}_L}{s} \qquad (8)$$

where

$$\overline{\mathbb{H}}_C = \begin{bmatrix} \alpha_1 & 0 & -\alpha_1 & 0 \\ 0 & \alpha_1 & 0 & -\alpha_1 \\ -\alpha_1 & 0 & \alpha_1 + \frac{\gamma_1+\gamma_2}{2} & -\frac{\gamma_2-\gamma_1}{2} \\ 0 & -\alpha_1 & -\frac{\gamma_2-\gamma_1}{2} & \alpha_1 + \frac{\gamma_1+\gamma_2}{2} \end{bmatrix}$$

and

$$\overline{\mathbb{H}}_L = \begin{bmatrix} \beta_1 & 0 & -\beta_1 & 0 \\ 0 & \beta_1 & 0 & -\beta_1 \\ -\beta_1 & 0 & \beta_1 + \frac{\delta_2}{2} & -\frac{\delta_2}{2} \\ 0 & -\beta_1 & -\frac{\delta_2}{2} & \beta_1 + \frac{\delta_2}{2} \end{bmatrix}.$$

$\overline{\mathbb{H}}_C$ and $\overline{\mathbb{H}}_L$ are the residue matrices of $\overline{\mathbb{Y}}(s)$ at the poles $s = \infty$ and $s = 0$, respectively, and are non-negative-definite. $\overline{\mathbb{Y}}(s)$ satisfies $\overline{\mathbb{Y}}(s) = -\overline{\mathbb{Y}}^T(-s)$, and elements of $\overline{\mathbb{Y}}(s)$ are holomorphic in Re $s \geqq 0$ except the above poles. Thus $\overline{\mathbb{Y}}(s)$ is the reactance matrix in s.

From eqn.(8) we can obtain the following with respect to elements in Fig.1.

$$C_1 = \alpha_1$$

$$C_2 = \frac{\gamma_2 - \gamma_1}{2}$$

$$C_3 = \gamma_1$$

$$L_1 = \frac{1}{\beta_1}$$

$$L_2 = \frac{2}{\delta_2}.$$

Finally, we shall find $\widetilde{y}_{11}$ and $\widetilde{y}_{12}$ from eqn.(7). Thus,

$$\widetilde{y}_{11} = \frac{1}{2}\left[(a_1 + a_2) p + \frac{b_1 + b_2}{p}\right]$$

and

$$\widetilde{y}_{12} = \frac{1}{2}\left[(a_1 - a_2) p + \frac{b_1 - b_2}{p}\right].$$

The admittance matrix $\widetilde{\mathbb{Y}}(p) = [\widetilde{y}_{ij}(p)]$ of $\widetilde{\mathbb{N}}$ in Fig.2 becomes

$$\widetilde{\mathbb{Y}}(p) = p\widetilde{\mathbb{H}}_C + \frac{\widetilde{\mathbb{H}}_L}{p} \qquad (9)$$

where

$$\widetilde{\mathbb{H}}_C = \frac{1}{2}\begin{bmatrix} a_1 + a_2 & a_1 - a_2 \\ a_1 - a_2 & a_1 + a_2 \end{bmatrix}$$

and

$$\widetilde{\mathbb{H}}_L = \frac{1}{2}\begin{bmatrix} b_1 + b_2 & b_1 - b_2 \\ b_1 - b_2 & b_1 + b_2 \end{bmatrix}.$$

$\widetilde{\mathbb{H}}_C$ and $\widetilde{\mathbb{H}}_L$ are the residue matrices of $\widetilde{\mathbb{Y}}(p)$ at the poles $p = \infty$ and $p = 0$, respectively, and are non-negative-definite. $\widetilde{\mathbb{Y}}(p)$ satisfies $\widetilde{\mathbb{Y}}(p) = -\widetilde{\mathbb{Y}}^T(-p)$, and elements of $\widetilde{\mathbb{Y}}(p)$ are holomorphic in Re$p \geqq 0$ except the above poles. Thus the 2×2 matrix $\widetilde{\mathbb{Y}}(p)$ is the reactance matrix in p. Hence, we are at a position of realizing $\widetilde{\mathbb{Y}}(p)$ by using transmission lines.

To extract a coupled 2-wire line from the reactance matrix $\widetilde{\mathbb{Y}}(p)$ of eqn.(9), $\widetilde{\mathbb{Y}}(1)$ is calculated. The result is

$$\widetilde{\mathbb{Y}}(1) = \frac{1}{2}\begin{bmatrix} y_{o1} + y_{e1} & y_{o1} - y_{e1} \\ y_{o1} - y_{e1} & y_{o1} + y_{e1} \end{bmatrix} \qquad (10)$$

where

$$y_{o1} = a_1 + b_1$$

and

$$y_{e1} = a_2 + b_2.$$

Symbols y_{e1} and y_{o1} are the even and odd mode characteristic admittances of the extracted 2-wire line.

Let $\widetilde{\mathbb{Y}}_{\mathbf{R}}(p)$ be the remainder after extracting the above coupled 2-wire line from $\widetilde{\mathbb{Y}}(1)$, then

$$\widetilde{\mathbb{Y}}_{\mathbf{R}}(p) = \widetilde{\mathbb{Y}}(1)\left[\widetilde{\mathbb{Y}}(1) - p\widetilde{\mathbb{Y}}(p)\right]^{-1}\left[\widetilde{\mathbb{Y}}(p) - p\widetilde{\mathbb{Y}}(1)\right].$$

From theorem 1, $\widetilde{\mathbb{Y}}_{\mathbf{R}}(p)$ is the 2×2 reactance matrix in p and is given by

$$\widetilde{\mathbb{Y}}_{\mathbf{R}}(p) = \frac{1}{2p}\begin{bmatrix} y_{o2}+y_{e2} & y_{o2}-y_{e2} \\ y_{o2}-y_{e2} & y_{o2}+y_{e2}\end{bmatrix} \quad (11)$$

where

$$y_{o2} = \frac{b_1}{a_1} y_{o1}$$

and

$$y_{e2} = \frac{b_2}{a_2} y_{e1}.$$

$\widetilde{\mathbb{Y}}_{\mathbf{R}}(p)$ may be realized by taking a connection where both wires of the coupled line are ground-connected at the far end. Here y_{o2} and y_{e2} are two mode characteristic admittances of this 2-wire line. We complete the proof of theorem 2. Q.E.D.

Theorem 3: Necessary and sufficient conditions for a reactance 2×2 symmetric matrix $\mathbb{W}(p,s) = [w_{ij}(p,s)]$ to be realized as the admittance matrix of the 2-port network between ports 1 and 2 in Fig.1(b) are that

(1) $w_{11}(p,s) = w_{22}(p,s)$

(2) for $k = 1, 2$,

$$\frac{1}{w_{11}(p,s) \pm w_{12}(p,s)} = \alpha_k s + \cfrac{1}{\beta_k s + \cfrac{\gamma_k}{s} + a_k p + \cfrac{b_k}{p}} \quad (12)$$

where (i) $\alpha_1 = \alpha_2 > 0$, $\beta_1 = \beta_2 > 0$, $\gamma_2 > \gamma_1 > 0$, (ii) $a_1 + b_1 > a_2 + b_2 > 0$, $a_2 b_1 (a_1 + b_1) > a_1 b_2 (a_2 + b_2) > 0$ for $a_k, b_k > 0$.

When "+", $k = 1$ must be adapted, and when "−", $k = 2$.

Proof: The proof of necessity will be omitted here. Therefore, only the sufficiency is proved. Eqn.(12) can be written as

$$w_{11}(p,s) \pm w_{12}(p,s) = \frac{1}{\alpha_k s} - \frac{\left(\frac{1}{\alpha_k s}\right)^2}{\beta_k s + \left(\gamma_k + \frac{1}{\alpha_k}\right)\frac{1}{s} + a_k p + \frac{b_k}{p}}. \quad (13)$$

By comparing eqn.(13) with eqn.(1), we can get following relations:

$$\bar{y}_{11} \pm \bar{y}_{12} = \frac{1}{\alpha_k s} \quad (14)$$

$$\bar{y}_{13} \pm \bar{y}_{14} = \varepsilon \frac{1}{\alpha_k s} \quad (\varepsilon = \pm 1) \quad (15)$$

$$\bar{y}_{33} \pm \bar{y}_{34} = \beta_k s + \left(\gamma_k + \frac{1}{\alpha_k}\right)\frac{1}{s} \quad (16)$$

$$\widetilde{y}_{11} \pm \widetilde{y}_{12} = a_k p + \frac{b_k}{p} \quad (17)$$

From eqn.(14) we have

$$\bar{y}_{11} = \frac{1}{\alpha_1 s}$$

and

$$\bar{y}_{12} \equiv 0.$$

To obtain $\bar{y}_{13}$ and $\bar{y}_{14}$ from eqn.(15), we will adopt $\varepsilon = -1$. Then,

$$\bar{y}_{13} = -\frac{1}{\alpha_k s}$$

and

$$\bar{y}_{14} \equiv 0.$$

From eqn.(16) ,

$$\bar{y}_{33} = \beta_1 s + \left(\frac{1}{\alpha_1} + \frac{\gamma_1 + \gamma_2}{2}\right)\frac{1}{s}$$

and

$$\bar{y}_{34} = -\frac{\gamma_2 - \gamma_1}{2}\frac{1}{s}.$$

Thus the 4×4 admittance matrix $\overline{\mathbb{Y}}(s)$ of $\overline{\mathbb{N}}$ in Fig.2 may be written as

$$\overline{\mathbb{Y}}(s) = \frac{\overline{\mathbb{H}}_{L1}}{s} + \overline{\mathbb{H}}_C s + \frac{\overline{\mathbb{H}}_L}{s} \quad (18)$$

where

$$\overline{\mathbb{H}}_{L1} = \begin{bmatrix} \frac{1}{\alpha_1} & 0 & -\frac{1}{\alpha_1} & 0 \\ 0 & \frac{1}{\alpha_1} & 0 & -\frac{1}{\alpha_1} \\ -\frac{1}{\alpha_1} & 0 & \frac{1}{\alpha_1} + \frac{\gamma_2-\gamma_1}{2} & -\frac{\gamma_2-\gamma_1}{2} \\ 0 & -\frac{1}{\alpha_1} & -\frac{\gamma_2-\gamma_1}{2} & \frac{1}{\alpha_1} + \frac{\gamma_2-\gamma_1}{2}\end{bmatrix}$$

$$\overline{\mathbb{H}}_C = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \beta_1 & 0 \\ 0 & 0 & 0 & \beta_1 \end{bmatrix}$$

and

$$\overline{\mathbb{H}}_L = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \gamma_1 & 0 \\ 0 & 0 & 0 & \gamma_1 \end{bmatrix}.$$

$\overline{\mathbb{H}}_{L1}$, $\overline{\mathbb{H}}_C$ and $\overline{\mathbb{H}}_L$ are non-negative definite residue matrices. Thus it is easy to see that $\overline{\mathbb{Y}}(s)$ is the reactance matrix in s. From eqn.(18), we obtain the following element values.

$$L_1 = \alpha_1$$

$$L_2 = \frac{2}{\gamma_2 - \gamma_1}$$

$$L_3 = \frac{1}{\gamma_1}$$

$$C_3 = \beta_1.$$

We are at a position of realizing $\widetilde{\mathbb{Y}}(p)$ by using 2-wire lines. Find $\widetilde{y}_{11}$ and $\widetilde{y}_{12}$ from eqn.(17), then we can have

$$\widetilde{y}_{11} = \frac{1}{2}\left[(a_1 + a_2)\,p + \frac{b_1 + b_2}{p}\right]$$

and

$$\widetilde{y}_{12} = \frac{1}{2}\left[(a_1 - a_2)\,p + \frac{b_1 - b_2}{p}\right].$$

Thus the admittance matrix $\widetilde{\mathbb{Y}}(p) = [\widetilde{y}_{ij}(p)]$ of $\widetilde{\mathbb{N}}$ in Fig.2 becomes

$$\widetilde{\mathbb{Y}}(p) = p\widetilde{\mathbb{H}}_C + \frac{\widetilde{\mathbb{H}}_L}{p}$$

where

$$\widetilde{\mathbb{H}}_C = \frac{1}{2}\begin{bmatrix} a_1 + a_2 & a_1 - a_2 \\ a_1 - a_2 & a_1 + a_2 \end{bmatrix}$$

and

$$\widetilde{\mathbb{H}}_L = \frac{1}{2}\begin{bmatrix} b_1 + b_2 & b_1 - b_2 \\ b_1 - b_2 & b_1 + b_2 \end{bmatrix}.$$

The matrix $\widetilde{\mathbb{Y}}(p)$ defined above is the same into eqn.(9). Hence, $\widetilde{\mathbb{Y}}(p)$ is realized as the cascade of coupled two 2-wire lines, and both wires are ground-connected at the far end. As the result, we see the even and odd mode characteristic impedances are the same values that were described in theorem 2. Q.E.D.

4 Example

Let us illustrate the application of theorems to the synthesis technique presented by considering an example.

Realize a reactance matrix $\mathbb{W}(p,s) = [w_{ij}(p,s)]$ $(i,j = 1,2)$ where $w_{ij}(p,s)$'s are given by

$$w_{11}(p,s) = w_{22}(p,s) = \frac{2\left\{\begin{array}{c} 48s^2p^4 + 12s\left(27s^2+26\right)p^3 \\ +\left(216s^4 + 379s^2 + 120\right)p^2 \\ +2s\left(15s^2+13\right)p + s^2 \end{array}\right\}}{\begin{array}{c} s\left\{3sp^2 + 6\left(3s^2+4\right)p + s\right\} \\ \times\left\{16sp^2 + 12\left(s^2+1\right)p + s\right\} \end{array}}$$

and

$$w_{12}(p,s) = w_{21}(p,s) = \frac{4p\left(42sp^2 - 12p + s\right)}{\begin{array}{c} s\left\{3sp^2 + 6\left(3s^2+4\right)p + s\right\} \\ \times\left\{16sp^2 + 12\left(s^2+1\right)p + s\right\} \end{array}}.$$

First of all, let us calculate $w_{11}(p,s) \pm w_{12}(p,s)$. The results are

$$\frac{1}{w_{11}(p,s) + w_{12}(p,s)} = \frac{1}{2}s + \frac{1}{3s + \dfrac{1}{s} + 4p + \dfrac{1}{4p}}$$

and

$$\frac{1}{w_{11}(p,s) - w_{12}(p,s)} = \frac{1}{2}s + \frac{1}{3s + \dfrac{2}{s} + \dfrac{1}{2}p + \dfrac{1}{6p}}.$$

The above two expressions suggest the application possibility of theorem 3. According to eqn.(18), we can have the following:

$$\alpha_1 = \alpha_2 = \frac{1}{2}$$

$$\beta_1 = \beta_2 = 3$$

$$\gamma_1 = 1, \quad \gamma_2 = 2$$

$$a_1 = 4, \quad a_2 = \frac{1}{2}$$

$$b_1 = \frac{1}{4}, \quad b_2 = \frac{1}{6}.$$

For the above coefficients, we see that (i), (ii) in theorem 3 are satisfied. From eqn.(18) the admittance matrix $\overline{\mathbb{Y}}(s)$ of $\overline{\mathbb{N}}$ in Fig.2 is given by

$$\overline{\mathbb{Y}}(s) = \begin{bmatrix} \frac{2}{s} & 0 & -\frac{2}{s} & 0 \\ 0 & \frac{2}{s} & 0 & -\frac{2}{s} \\ -\frac{2}{s} & 0 & 3s + \frac{7}{2s} & -\frac{1}{2s} \\ 0 & -\frac{2}{s} & -\frac{1}{2s} & 3s + \frac{7}{2s} \end{bmatrix}.$$

Thus LC element values in Fig.1(b) are given by

$$L_1 = \frac{1}{2},\ L_2 = 2,\ L_3 = 1,\ \text{and}\ C_3 = 3.$$

Finally, let us consider the realization of $\widetilde{\mathbb{N}}$ in Fig.2. From eqn.(9), the corresponding admittance matrix $\widetilde{\mathbb{Y}}(p)$ can be written as

$$\widetilde{\mathbb{Y}}(p)=\frac{1}{24p}\begin{bmatrix} 54p^2+5 & 42p^2+1 \\ 42p^2+1 & 54p^2+5 \end{bmatrix}.$$

Thus, as shown from eqn.(10), two mode characteristic admittances y_{o1} and y_{e1} of the coupled line which can be extracted from $\widetilde{\mathbb{Y}}(p)$ are

$$y_{o1} = \frac{17}{4},\ \text{and}\ y_{e1} = \frac{2}{3}.$$

Theorem 1 is applied to $\widetilde{\mathbb{Y}}(p)$ to obtain the admittance matrix $\widetilde{\mathbb{Y}}_R(p)$ after the 2-wire line having the characteristic admittances above was extracted. Thus,

$$\widetilde{\mathbb{Y}}_R(p) = \frac{1}{1152p}\begin{bmatrix} 281 & 25 \\ 25 & 281 \end{bmatrix}.$$

The above is the admittance matrix of 2-port network where both wires of a 2-wire line are ground-connected at the far end. Thus, the odd- and even-mode characteristic admittances become

$$y_{o2} = \frac{17}{64},\ \text{and}\ y_{e2} = \frac{2}{9},$$

respectively.

5 Conclusion

Necessary and sufficient conditions have been presented under which a 2-variable 2x2 admittance matrix may be realized as a 2-variable 2-port network in which two distinct 4-port networks consisting of LC elements is terminated in a cascade of coupled 2-wire lines.

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