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Study of Boundary Layer Control of Divergent Flow with Shock Wave

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In this paper, it aimed to clarify flow situation in which a supersonic jet, issuing from a divergent nozzle with shock wave, is changed by difference of stagnation pressure and nozzle angle. Angles of the divergent nozzles were 10 degrees and 20 degrees respectively, and the study was done by experiment and numerical analysis. Pressure ratio *Pd/Pa* was from 1.0 to 6.0 for the experiment, and it was 1.0, 3.0, 6.0, and 10.0 for the numerical analysis. In the experiment, total pressure vibrations and frequency analysis of total pressure vibrations were measured in the vicinity of the nozzle exit. In the numerical analysis, it was examined about velocity contours and pressure contours. Especially, in the experiment, they were confirmed from a result of frequency analysis of total pressure vibrations that the separation point exerted on a jet boundary layer and the flow exerted on self-excited vibration.

At the result, according to difference of nozzle's divergent angle and pressure ratio, remarkable differences were seen in the behavior of separation points, the flow behavior of downstream region, and the flow situation in the vicinity of nozzle exit. Moreover, it is clarified in high pressure ratio that flow became a supersonic flow with pseudo-shock wave repeating overexpansion and overcompression.

Keywords : Compressible flow, Separation, Boundary layer, Numerical analysis,
Divergent nozzle

1. Introduction

A supersonic jet flow issuing from a divergent nozzle is widely used in the industrial field which is related to an aerospace field [1] according to jet propulsion and minute powder production of metal and nonmetal [2]. It is very important in the industry to clarify the flow which exists in divergent nozzles, in the vicinity of their exit, and in the downstream region of the flow. However, the flow has not been clarified enough because it is complex and to examine it by experiment and numerical analysis is very difficult. Especially, a research is hardly seen on the unsteady supersonic flow with shock wave [3].

It have been aimed by authors to control the supersonic jet flow issuing from a divergent nozzle by the inhalation and exhalation of control air from the control holes which are installed in nozzle's divergent part for research and development of special industrial nozzles. In a word, it has aimed to control the separation of a flow in the nozzle's divergent part by inhalation and exhalation of control flows, and to clarify the influence on the boundary layer from a supersonic jet. In already reported [4], the position of the separation point of the flow caused in the nozzle's divergent part, and the influence that the presence of the control flow and the difference of stagnation pressure exerted on the boundary layer of the jet were clarified in the subsonic flow and the supersonic flow.

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A supersonic jet controlled by inhalation and exhalation of control flows will be described in the next report. In this paper, it is studied that the nozzles' divergent angle and the difference of stagnation pressure influence the flow field of a supersonic, with shock wave, issuing from a divergent nozzle. In this time, the change of separation points and the behavior of flows are clarified by the experiment and the numerical analysis with the nozzle whose divergent part length is longer than the already reported nozzle. In addition, the frequency analysis of total pressure of a flow field was done by the experiment, and it was studied that controlling separation points influenced a boundary layer and self-excited vibration of a jet.

2. Experiment method and Analysis method

In the Fig. 1, an outline chart of experiment equipments is shown. The air fed forcefully from an oilfree compressor is continuously issuing from a divergent nozzle installed in stagnation tanks to a room of absorbing sound.

In the Fig. 2, a nozzle shape used by the experiment and the numerical analysis is shown. In the Table 1, various sizes of a nozzle are shown. In this time, two kinds of nozzles with different divergent angles are used, and one whose divergent angle is 20° was set as nozzle 1 and the other whose divergent angle is 10° was set as nozzle 2. Although eight control holes to control a divergent jet have been installed, there was no inhalation and exhalation of air from the control holes and the control flow was not used in this paper. The nozzle's center axis was set as X axis and the position of the nozzle exit was set as $X=0$.

In the numerical analysis, velocity contours and pressure contours were done by pressure ratio Po/Pa, which was stagnation pressure Po divided by atmospheric pressure Pa, =1.0, 3.0, 6.0, and 10.0. The momentum conservation (Navier-Stokes equation), the equation of continuity, the energy conservation, and the finite volume method are as primitive equations.

In the experiment, it was done in the region of pressure ratio Po/Pa from 1.0 to 6.0. The total pressure vibrations was measured by using the pressure transducer, which is a micro semiconductor type, was built into in a total pressure probe as shown in Fig. 3. The frequency analysis of total pressure was done with the fast Fourier transform analyzer.

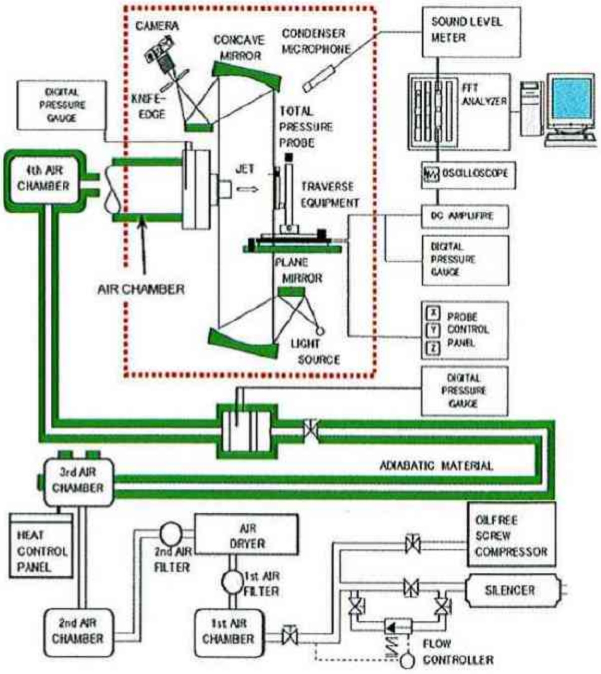


Fig. 1 Experiment equipment

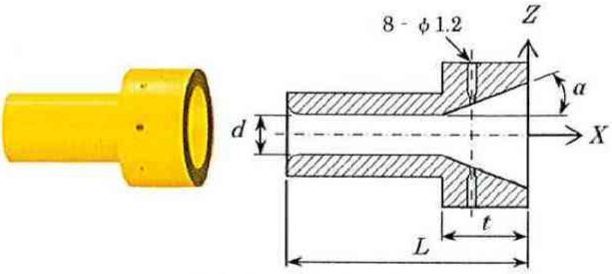


Fig. 2 Nozzle shape

Table 1 Configuration of divergent nozzles

Symbol	L (mm)	d (mm)	t (mm)	α (°)
Nozzle1	56	9.2	20	20
Nozzle2	56	9.2	20	10

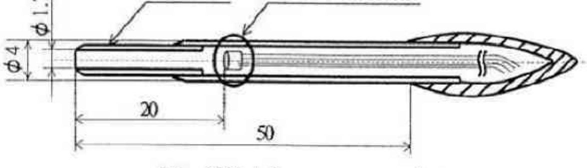


Fig. 3 Total pressure probe

3. Results and consideration

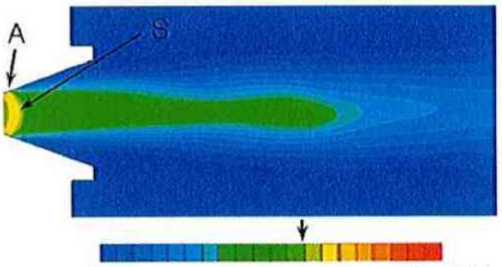
3.1 Results of nozzle 1

In the Fig. 4(a), 4(b), the velocity contours and the pressure contours of $Po/Pa=1.0$ in nozzle 1 are shown. In the gauge bar of velocity contours, arrow ↓ is fixed to the position that corresponds to sound speed.

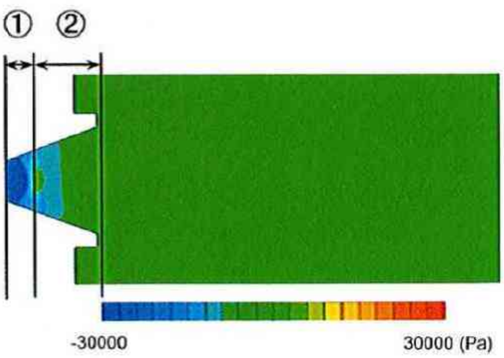
As showing in the Fig. 4(a), it became a supersonic flow in nozzle 1 because the flow expanded rapidly in the divergent part. A weak round shock wave S was generated. In the rearward, the flow decelerated to subsonic sound and diffused. It became a flow which decelerates uneasily in the center part of the jet flow. The separation of the flow appeared in the vicinity of starting point A on the inside of the divergent part of the nozzle. From the analytical result at the cycle time, it was guessed that the jet flow vibrated intensely because the separation point changed intensely in the vicinity of A.

As showing in the Fig. 4(b), the pressure changing discretely was seen in the region of the vicinity of the starting divergence ① of the nozzle inside for a round shock wave. In the back region ②, it became pressure contours of vertical stripes in the cross section of the divergent part. In the inside of the flow issuing from the nozzle, some regions of the pressure increasing and decreasing were seen and the flow diffused to the atmospheric pressure.

As in Fig. 5, the frequency analysis of the total pressure of $Po/Pa=1.2$ in nozzle 1 is shown. The discrete frequency distribution that has peaks was seen in the vicinity of $X/d=0.0$-1.5 on the center axis of the jet. The fundamental frequency of the first peak is caused in about 3.6kHz and from the second peak is generated at the position of multiple of fundamental frequency. It is thought that these peaks originated from the fact that the separation point vibrated intensely by the self-excited vibration of the flow of the divergent part. The thumping vibrations of the separation point was confirmed from the numerical analysis result at the cycle time of the velocity contours in the Fig. 4(a) and it was guessed that the jet vibrated intensely for the reason. It was guessed



(a) Velocity contour



(b) Pressure contour

Fig.4 Velocity and pressure contours for $Po/Pa=1.0$, Nozzle 1

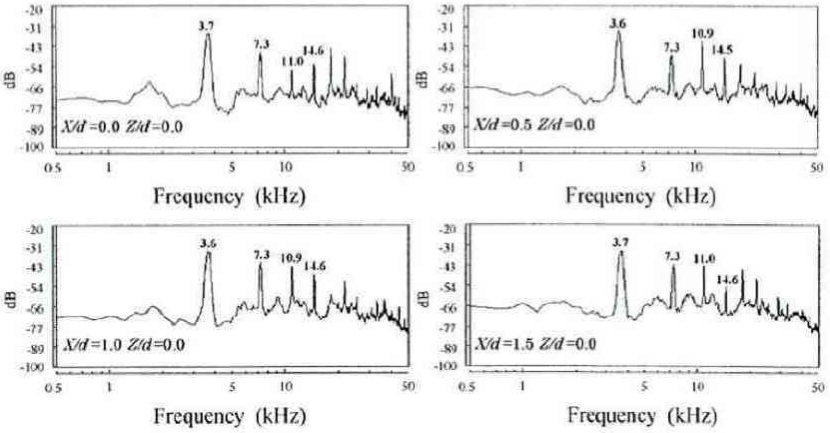


Fig.5 Frequency analysis of total pressure for $Po/Pa=1.2$. Nozzle 1

that the thumping vibrations of the separation point were promoted by the self-excited vibration of the jet more. In the result from doing frequency analysis of noise, it has been understood that the frequency of total pressure vibrations and the location of occurrence of peaks are almost corresponding. From this, it was guessed that the self-excited vibration greatly originated as a source of noise.

In the Fig. 6(a), 6(b), the velocity contours and the pressure contours of $Po/Pa=3.0$ in nozzle 1 are shown.

As showing in the Fig. 6(a), the separation point A of the flow moved to the downstream side more than $Po/Pa=1.0$ because the stagnation pressure increased, and the shock wave moved to the downstream more, too. Weak slipping side Ls were seen behind the shock wave. Moreover, the jet was a stable flow compared with the case of $Po/Pa=1.0$.

As showing in the Fig. 6(b), the discretely pressure change was seen in the vicinity of the starting divergence of the nozzle and the area ③ in the nozzle divergent part.

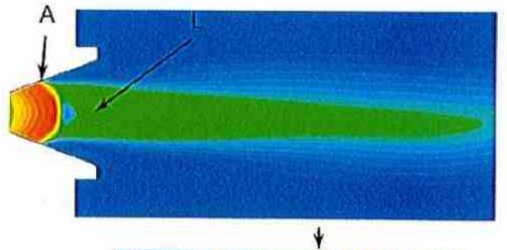
In the Fig. 7(a), 7(b), the velocity contours and the pressure contours of $Po/Pa=6.0$ in nozzle 1 are shown.

As showing in the Fig. 7(a), the separation point A moved to the downstream side further than $Po/Pa=3.0$. A Mach disk M that grew up considerably in the vicinity of the nozzle exit was generated, and slipping side Ls existed to the downstream region considerably, too. Moreover, it was a subsonic flow in the area enclosed by the Mach disk and the slipping side. From a measurement result of the total pressure vibrations by the experiment, it became a flow with little vibrations comparatively. In the nozzle, which was already reported in (4), with a short length of the nozzle divergence, the separation was caused in the top of that nozzle. That was a steady flow with little vibrations than nozzle 1 with a long length of the nozzle divergence.

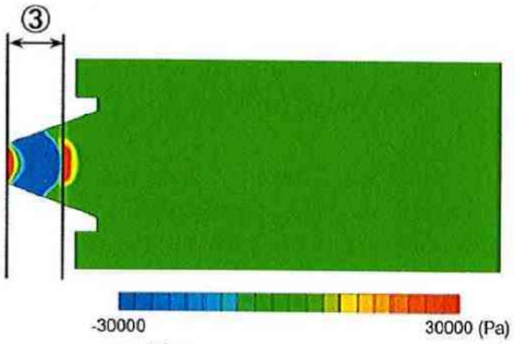
As showing in the Fig. 7(b), the discontinuous pressure distribution corresponding to shock wave was seen in the area ④ of the nozzle divergent part as well as the case of $Po/Pa=3.0$.

In the Fig. 8(a), 8(b), the velocity contours and the pressure contours of $Po/Pa=10.0$ in nozzle 1 are shown.

As showing in the Fig. 8(a), the flow was accelerated in the divergent part of the nozzle, and it became a supersonic flow in the entire region of the nozzle inside. The flow separated in the end E of the nozzle, and a Mach disk M which was plat in the center part

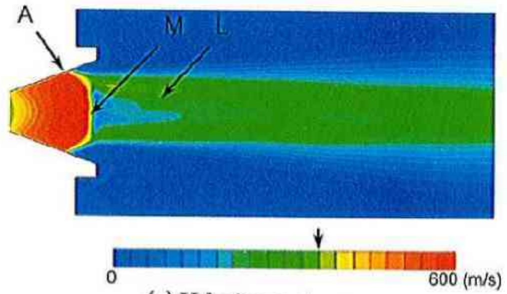


(a) Velocity contour

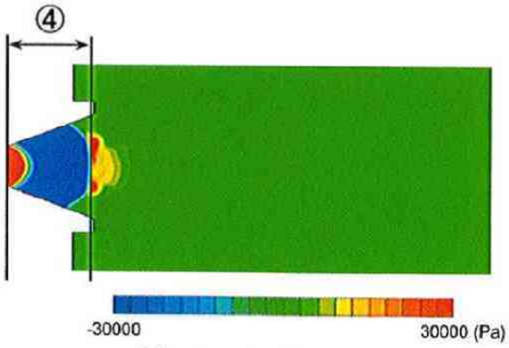


(b) Pressure contour

Fig.6 Velocity and pressure contours for $Po/Pa=3.0$, Nozzle 1



(a) Velocity contour



(b) Pressure contour

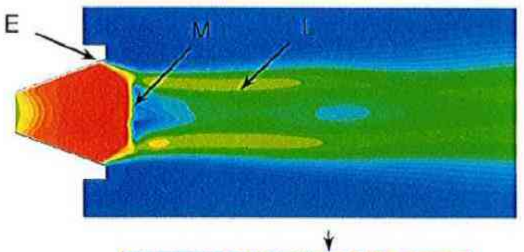
Fig.7 Velocity and pressure contours for $Po/Pa=6.0$, Nozzle 1 and grew up considerably was formed behind the oblique shock wave caused from the end of the nozzle. Slipping side Ls were seen in the rearward, and it became a subsonic flow.

As showing in the Fig. 8(b), a discontinuous pressure change was seen in the region ⑤, and there was a region where pressure increased because it decelerated to subsonic speed rapidly in the narrow area ⑥ behind the Mach disk. In the downstream region, it decelerated to subsonic speed while repeating the increase and decrease of pressure.

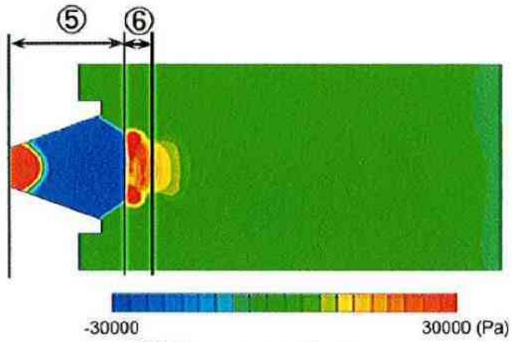
3.2 Results of nozzle 2

In the Fig. 9(a), 9(b), the velocity contours and the pressure contours of *Po*/*Pa* =1.0 in nozzle 2 are shown.

As showing in the Fig. 9(a), in nozzle 2 with a small divergent angle, the flow separated just after the divergent of the nozzle at A and B. Because the se-

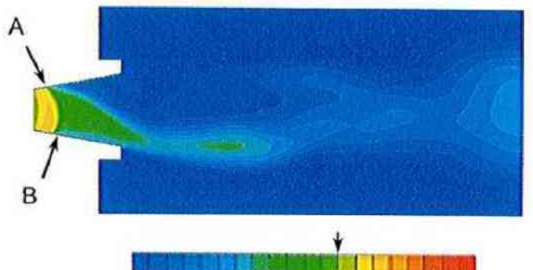


(a) Velocity contour

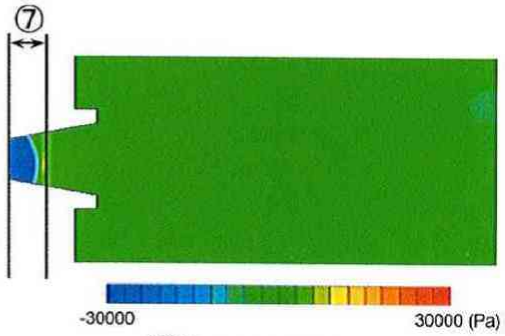


(b) Pressure contour

Fig.8 Velocity and pressure contours for *Po* /*Pa* =10.0, Nozzle 1



(a) Velocity contour



(b) Pressure contour

Fig.9 Velocity and pressure contours for *Po* /*Pa* =1.0, Nozzle 2

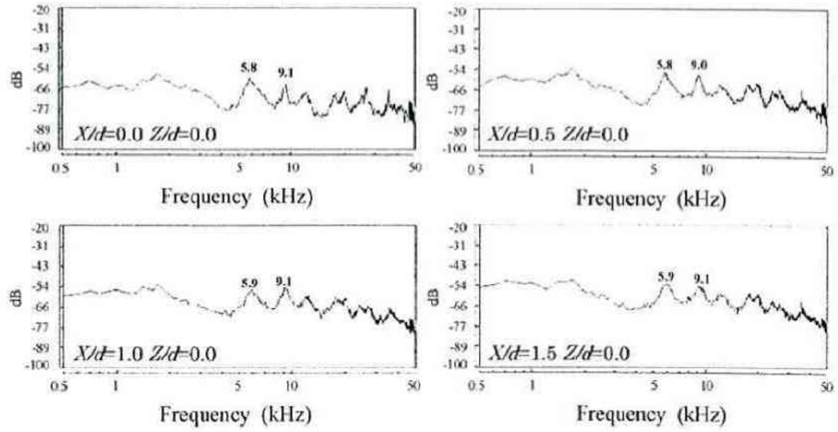


Fig.10 Frequency analysis of total pressure for *Po* /*Pa* =1.2, Nozzle 2

parathion point A and B were different up and down and the pressure difference was generated, according to the Coanda phenomenon, it became a flow that adhered to the nozzle wall, and vibrated greatly in the downstream region behind the nozzle.

As showing in the Fig. 9(b), in the region ⑦ where the flow separated, the pressure change was discontinuous. In the rearward, the area where the pressure increases and decreases was seen according to the vibrations of the flow.

In the Fig. 10, the frequency analysis of total pressures of $Po/Pa=1.2$ in nozzle 2 is shown. The discrete frequency distribution that has peaks was seen in the vicinity of $X/d=0.0$-1.5 on the center axis of the jet. Compared with nozzle 1 having a large divergent angle 20° shown in Fig. 5, the peak value of ones whose fundamental frequency greatly appeared were small. Therefore, the noise level of nozzle 1 appeared high though it was not shown here.

In the Fig. 11(a), 11(b), the velocity contours and the pressure contours of $Po/Pa=3.0$ in nozzle 2 are shown.

As showing in the Fig. 11(a), the flow separated at A before the nozzle exit and a shock wave was generated. Slipping side Ls were caused in the rearward.

As showing in the Fig. 11(b), in the area ⑧ corresponding to shock wave, the pressure change was discontinuous. The change of the pressure distribution was not seen in the downstream region behind the nozzle. The flow became a steady flow whose vibrations were little.

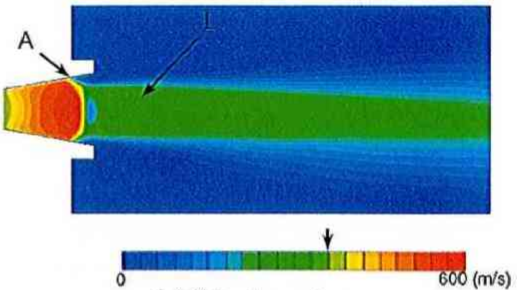
In the Fig. 12(a), 12(b), the velocity contours and the pressure contours of $Po/Pa=6.0$ in nozzle 2 are shown.

As showing in the Fig. 12(a), the flow separated in the exit E of the nozzle and a small Mach disk M was seen. In the rearward, it decelerated to subsonic speed while repeating overexpansion and overcompression.

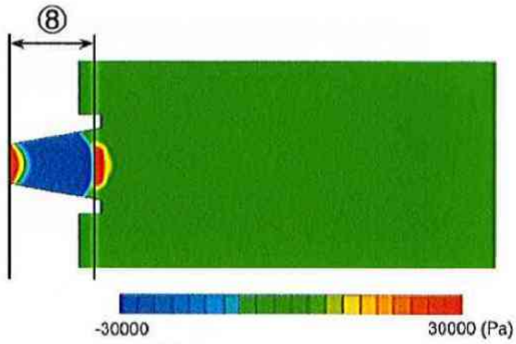
As showing in the Fig. 12(b), it is understood that it is a flow with a pseudo-shock wave from repeating overexpansion and overcompression.

In the Fig. 13(a), 13(b), the velocity contours and the pressure contours of $Po/Pa=10.0$ in nozzle 2 are shown.

As showing in the Fig. 13(a), it became a flow repeating clear overexpansion and clear overcompression with a pseudo-shock wave. For the reason, it became a flow that the supersonic speed considerably continued to the downstream region.

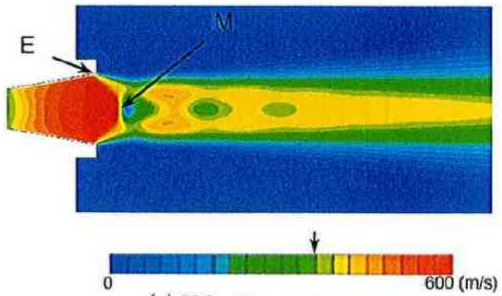


(a) Velocity contour

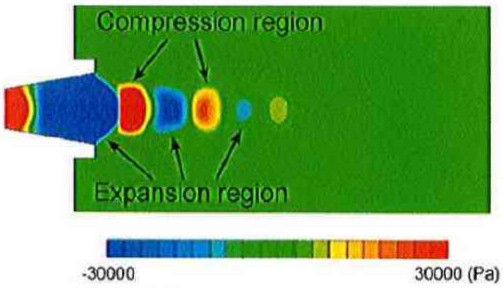


(b) Pressure contour

Fig.11 Velocity and pressure contours for $Po/Pa=3.0$, Nozzle 2



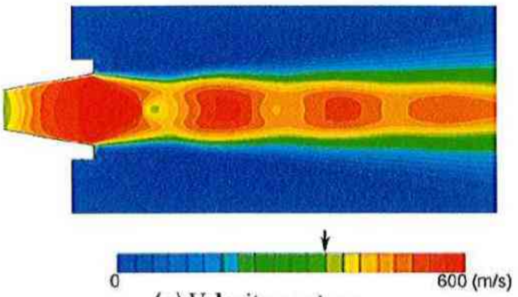
(a) Velocity contour



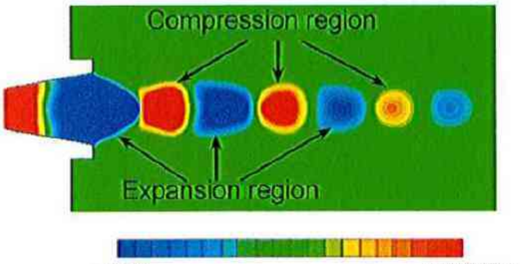
(b) Pressure contour

Fig.12 Velocity and pressure contours for $Po/Pa=6.0$, Nozzle 2

As showing in the Fig. 13(b), it is understood that the clearly overexpaning and overcompressing region where the increase and decrease of pressure was repeated was considerably formed to the downstream region and the flow was maintained to the supersonic speed.



(a) Velocity contour



(b) Pressure contour

Fig.13 Velocity and pressure contours for Po/Pa =10.0, Nozzle 2

4. Consideration about experiment results and CFD results

The purpose is to control a flow with a shock wave issuing from a divergent nozzle according to the experiment and the numerical analysis. Especially, the influence that the geometrical shape of the divergent nozzle and the difference of stagnation pressure exerted on the jet flow was examined in this time. In the experiment, measuring total pressure vibrations and analyzing frequency made comparative study with the result of the frequency analysis of the noise. In this time, a comparative study was made by the numerical analysis results of the unsteady flow in low pressure ratio and the experimental results by the self-excited vibration of the flow. As a result, in the vicinity of pressure ratio Po/Pa =1.2 which is comparatively low, the flow was with discrete frequency that has peaks having intense total pressure vibrations. By the numerical analysis results from unsteady analysis, it became a flow having vibrations, too. It is turned out that the experiment result comparatively corresponded with the numerical analysis results. From the results of the above-mentioned experiment and analysis, it is turned out that the vibrations of the separation point and the total pressure vibrations from the flow in the nozzle divergent part greatly influenced the self-excited vibration of the flow. In addition, in the future a more detailed flow is clarified by improving the accuracy of numerical analysis of an unsteady flow.

5. Conclusion

To the nozzles with a different divergent angle which is 10° and 20°, the experiment and the numerical analysis were done with changing stagnation pressure. The influence from the divergent part to the flow separation and the flow situation of rearward was examined. The following conclusions were obtained.

(1) It is turned out that the flow which generates vibrations in the unsteady numerical analysis is the same as the flow which generates vibrations of total pressure in the experiment.

(2) In the vicinity of stagnation pressure ratio Po/Pa =1.2, the result is almost corresponding between frequency analysis of total pressure vibrations and frequency analysis of noise.

(3) In the stagnation pressure ratio Po/Pa =6, the flow separates in the end of the nozzle which is with short divergent length and divergent angle 20° shown already reported and becomes a flow with a little vibrations than in the nozzle with long divergent length shown by this report.

(4) The separation point of a flow moves to the downstream side while increasing stagnation pressure ratio with which nozzle having a different divergent angle. There is a tendency that the separation point moves to downstream side more than the other when the nozzle is with a small divergent angle 10°.

(5) From the nozzle with a small divergent angle, it becomes a flow which generates a Coanda phenomenon attaching to the divergent part of the nozzle in the stagnation pressure ratio Po/Pa = 1.

(6) The self-excited vibration of the flow is greatly influenced from the vibrations of separation point and the total pressure vibrations.

(7) When stagnation pressure ratio becomes large, it becomes a flow which generates a clear Mach disk and slipping sides from the nozzle with a large divergent angle, and it becomes a supersonic flow with a pseudo-shock wave repeating overexpansion and overcompression from the nozzle with a

small divergent angle.

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