Mem. School. B.O.S.T. Kinki University No. 3 : 45~55 (1998)

Design and Development of a Binary Filter for Image Differentiation (No. 1)

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Abstract

The present study is intended to attempt differential operation for images in expectation of the opto-computing age to come by designing a digital filter for an analogue optical system.

When an object is illuminated with a monochromatic light to make its image, in general, its imaging optical system is called the double diffraction optical system.

Theories and experiments in the present study are all provided to be conducted with this double diffraction optical system and the light source with a coherent source such as laser light.

Well, the imaging optical system is easily treated if considered from the standpoint of transmission theory, and it often shows effective functions for various problems. In a coherently illuminated imaging system, when we consider amplitude distribution of the wave front that passed through the object as the input and that at the image plane as the output in comparison of the time axis in the transmission system with the spatial axis of the optical system, the optical system can be regarded as a kind of linear transform system. Thus, a variety of mathematical operations get possible between the amplitude transmittance distribution of the object and that of the image when a spatial filter is inserted, to actively vary characteristics of this transform system, on the spectral plane of the optical system, namely on Fraunhofer diffraction plane at the back focal plane just behind the object. In the present study, we discuss the principle of optical differential operation, a method of fabricating the optical differential filter necessary for it, and performances of the filter.

Introduction

Conducted here is design of a binary differential filter by utilizing the principle advocated by Lohmann et *al.*[1~3]. Now we fabricate a binary filter of an opaque plate that is divided into (m, n) square elements, on each of which we make an appropriate rectangular aperture, and insert it on the diffraction plane of a double diffraction optical system. When illuminating this with arbitrary plane waves, distribution of amplitude components of light waves at the image plane depends on area of the aperture, and distribution of its phase components on the relative shift from the center of each plane element to the center of each aperture. It is therefore possible to bring amplitude distribution of light waves at the image plane close to what is desired. Here we consider the structure of binary filter needed for amplitude distribution of light waves at the image plane corresponding just to the differential of amplitude transmittance distribution of the object.

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Besides, this method applies to no more than the differential filter as an approximation, and is effective only when amplitude transmittance distribution of the object takes nothing but binary value of either 0 or 1. We also discuss on the degree of approximation to some extent.

1. Principle

Figure 1 shows the imaging system used in the present study and its coordinate system. It is assumed that the optical system is totally free from aberration and that consideration is taken only on the imaging of paraxial rays. Accordingly, it is provided that isoplanatism has always held true. In the figure, (S_0) expresses an ideal source of coherent light, (P) the object plane, (Q) the spectral plane and (S_c) the image plane. Collimator, condenser and projection lenses are expressed as (L_1), (L_2), and (L_3), respectively. We employ conversion coordinates[4] of Eq. (1) to simplify correspondence of the object with its image, ease operation of Fourier conversion, and generalize treatment of the formula by making it dimensionless as mentioned later :

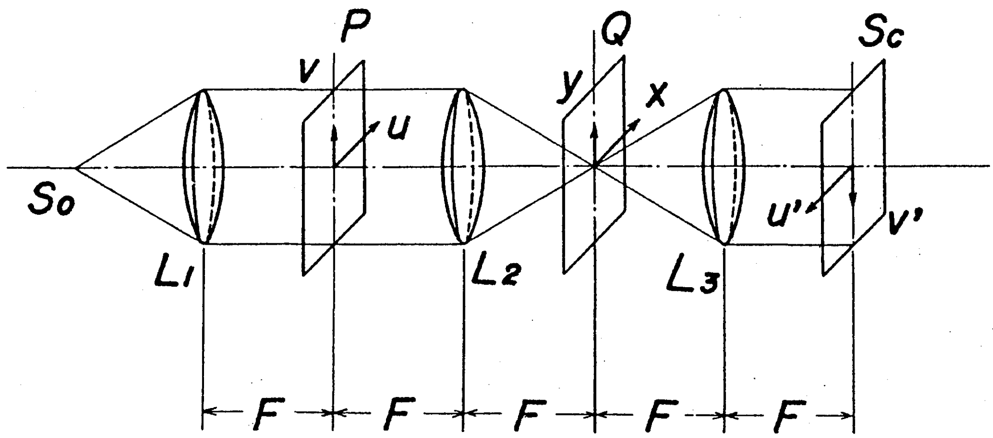

Figure 1 Double diffraction optical system and its coordinate system

$$
\left.
\begin{array}{ll}
u = (k \sin \delta) U, & v = (k \sin \delta) V \\
w = W & \\
u' = (k \sin \delta') U', & v' = (k \sin \delta') V' \\
x = X/A, & y = Y/A
\end{array}
\right\} \quad (1)
$$

Here, $k = 2\pi/\lambda$, and λ shows wavelength of the light source. Capital letters denote real ordinates and give actual length of geometry*). A denotes radius of projection lens, δ angular aperture of the connection system of projection and condenser lenses, and δ 'angular aperture of projection lens viewed from the image side. Denoted with (u, v) is a coordinate system in the object plane, and with (x, y) is that in the conjugate plane of the light source due to collimator and condenser lenses, namely in the Fraunhofer diffraction plane. Besides, (u', v') shows coordinate system in the image plane and w is the ordinate along the optical axis.

Provided the imaging magnification is M, it is given with

$$\frac{U'}{U} = \frac{V'}{V} = M \tag{2}$$

but use of the conversion coordinates, Eq.(1), is convenient because we can carry the dimensionless calculation as well as direct correspondence of the object and image at magnification of 1 .

Provided now that $f(u, v)$ expresses distribution of the complex amplitude transmittance of the object structure, distribution of the complex amplitude of the diffraction pattern on its Fraunhofer diffraction plane (x, y) is given by a Fourier conversion of $f(u, v)$, that is, by $o(x, y) = \mathscr{F}\{f(u, v)\}$. Similarly, distribution of the complex amplitude of the diffraction pattern on the wave front at the image plane (u', v') is given by Fourier conversion of $o(x, y)$, namely by $f'(u', v') = \mathscr{F}\{o(x, y)\}$. Therefore, because the condenser lens has an aperture so large as it passes all the wave front through and no aberration as well, amplitude distribution of the wave front at the image plane becomes $f'(u', v') \propto f(u, v)$ from Eqs.(1) and (2) after all. Here we leave to mention that an amplitude distribution of the object, same as geometrical optics**) (image of the object), at the image plane with Fourier conversions of the wave front twice, namely, the Fraunhofer diffraction. When imaging is conducted like this by two successive Fourier conversions, namely double diffraction, we call this imaging system as the double diffraction system, which is employed hereafter throughout the present paper.

2 . Design of a Binary Filter

Incidentally, a binary filter placed on the diffraction plane is divided into square elements having a side length of d_b as shown in Figure 2 .

*) From now on, conversion that requires no calculation of concrete values is to be conducted by means of this conversion coordinates.

**) Because directions of the coordinates (u, v) and (u', v') have been taken opposite as shown in Figure 1 , the image of $f'(u', v')$ has a shape that is inverted to the object $f(u, v)$ and agrees with imaging in geometrical optics.

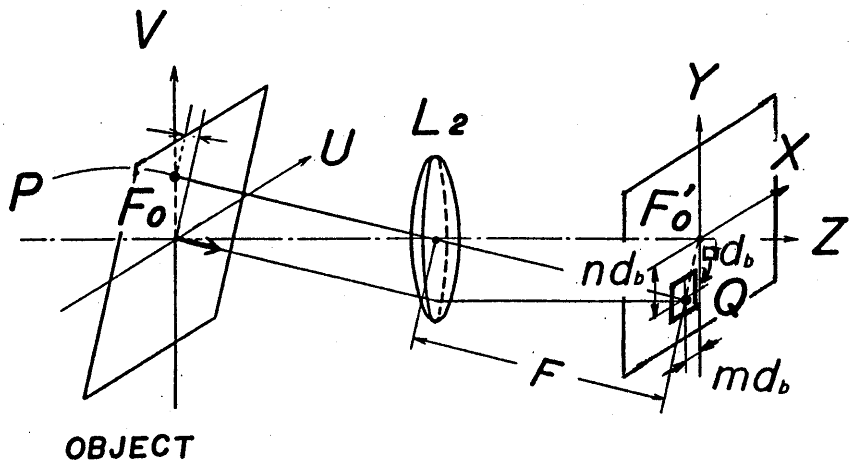

Figure 2 Object plane and Diffrection plane with a Filter

Now regarding the (m, n) th element, we consider an aperture (called cell, hereafter) with breadth of $b_{mn} \cdot d_b$ and height of d_b, and provide its trans-mittance as 1 whereas the remaining portion being 0. Besides, we provide that the coordinates of the center of this element are (md_b, nd_b) and that the cell center shifts from the element center by $p_{nm} \cdot d_b$ in the x direction. Here, b_{nm} and p_{mn} are parameters that depend on respective elements, $|P_{mn}| < 1/2$ and $0 \leq b_{nm} < 1$ in view of fabrication, and the whole size of the filter is determined by the spread of the diffraction pattern. By taking characte-ristics of the binary filter into consideration, in addition to the above, the following assumptions are provided :

i) Amplitude distribution of the image formed by light waves through the binary filter is confined within the square domain with side length of l'.

ii) Amplitude distribution of the diffraction pattern obtained by inverse Fourier conversion of that of the desired image is confined within the square domain with side length of w.

Now provided that the amplitude distribution of the desired image is denoted by $g'(u', v')$, that of the diffraction pattern obtained through its Fourier conversion becomes:

$$o(x,y) = const. \iint g'(u',v') \exp\{-j(u'x+v'y)\} du'dv' \tag{3}$$

Here, the relationships:

$$\begin{aligned} &g'(u', v') = 0, \ |u'| > l'/2, \ |v'| > l'/2 \\ &O(x, y) = 0, \ |x| > w/2, \ |y| > w/2 \end{aligned} \tag{4}$$

are provided to have been satisfied from the assumption written above.

Incidentally, the continuous function $o(x, y)$ is determined for every x and y if values of sampling points $x = m/l'$, $y = n/l'$ and $o(m/l', n/l')$ are given. This is usually called the sampling theorem in the frequency domain, and Eq.(3) is expressed as follows[5)]:

$$o(x, y) = \sum_m \sum_n o(m/l', n/l') \mathrm{sinc}(xl'-m) \cdot \mathrm{sinc}(yl'-n) \tag{5}$$

Here, the maximum spatial frequency contained in the function $o(x, y)$, namely the minimum divided value d_b, is given by:

$$\frac{d_b}{2} = \frac{1}{2l'} \tag{6}$$

In other words, $o(x, y)$ eventually proves to have been expressed perfectly as a set of $o(m/l', n/l')$, complex parameters represented by the group of sampling points $x = m/l'$, $y = n/l'$. Then, by knowing the amplitude distri-bution of the desired image $g'(u', v')$ again from Fourier conversion of Eq.(5),
and by expanding this into Fourier series, we obtain[*)]

$$\begin{aligned} g'(u', v') &= \frac{1}{l'^2} \iint o(x, y) \exp\{j(xu'+yu')\} dxdy \\ &= \frac{1}{l'^2} \cdot \mathrm{rect}(u'/l') \mathrm{rect}(v'/l') \sum_m \sum_m o(md_b, nd_b) \\ &\qquad \cdot \exp\{jd_b(mu'+nv')\} \end{aligned} \tag{7}$$

Incidentally, from the assumption of Eq.(4),

$$g'(u', v') \approx \sum_m \sum_n o(md_b, nd_b) \exp\{jd_b(mu'+nv')\} \tag{8}$$

holds true, so it eventually gives:

$$g'(u', v') \approx \sum_m \sum_n a_{mn} l^{j\varphi_{mn}/2\pi} \cdot \exp\{jd_b(mu'+nv')\} \tag{9}$$

if we denote a_{mn} and φ_{mn} for the amplitude term and phase term, respectively, for $o(md_b, nd_b)$ of each term with m and n in Eq.(8).

On the other hand, the transmittance function $S_b(x, y)$ of a binary filter composed of plane elements shown in Figure 3 is given by:

$$S_b(x, y) = \sum_m \sum_n \mathrm{rect}\left[\frac{x-(m+p_{mn})d_b}{b_{mn}d_b}\right] \mathrm{rect}\left[\frac{y-nd_b}{d_b}\right] \tag{10}$$

*) $\sum_m o(m/l') \mathrm{sinc}(l'x-m) \cdot \sum_n o(n/l') \mathrm{sinc}(l'y-n)$

$= (1/l')^2 \sum_m o(m/l') \int_{-l'/2}^{l'/2} \exp\{-ju'(x-m/l')\} du' \cdot \sum_n o(n/l') \int_{-l'/2}^{l'/2} \exp\{-jv'(y-n/l')\} dv'$

$= (1/l')^2 \sum_m \int_{-\infty}^{\infty} \mathrm{rect}(u'/l') o(m/l') \exp\{-ju'(x-md_b)\} du'$

$\cdot \sum_n \int_{-\infty}^{\infty} \mathrm{rect}(v'/l') o(n/l') \exp\{-jv'(y-nd_b)\} dv'$

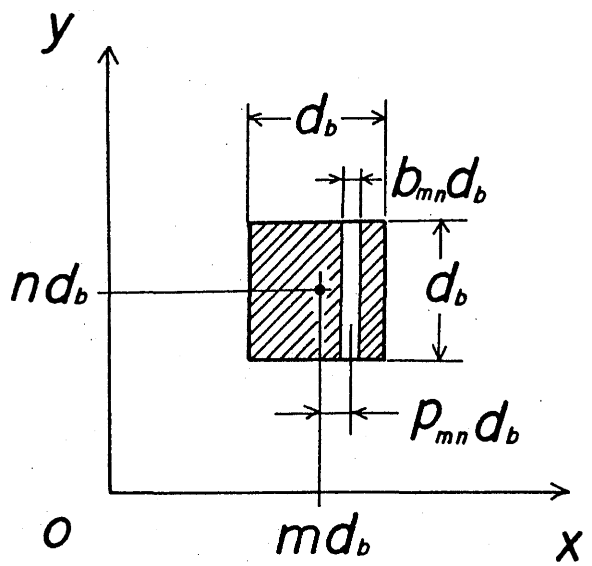

Figure 3 Binary filter composed of plane elements (m, n)

This filter is inserted on the diffraction plane (Q) shown in Figure 1, and illuminated by arbitrary plane waves that was emitted from a light source at a point u_o on the obiect plane (P) and passed through a lens (L_2). Under this condition, the amplitude distribution $h'(u, v')$ of wave front produced on the im age plane (S_c), by passing light waves after the filter again through the lens (L_3), is given by:

$$h'(u',\ v') = \int\int s_b(x,\ y)\exp j\{(u'+u_o)x+v'y\}\,dxdy \tag{11}$$

According to Eq.(10)

$$h'(u',\ v') = d^2\mathrm{sinc}(v'd_b)\sum_m \sum_n d_{mn}\mathrm{sinc}\{b_{mn}d_b(u'-u_o)\} \cdot\exp[jd_b\{(u'+u_o)(m+p_{mn})+v'n\}] \tag{12}$$

This formula may be regarded as the Fourier series expansion of the set (m, n) with respect to exp if the following approximations are satisfied:

$$\mathrm{sinc}\{b_{mn}d_b(u'+v_o)\} \approx const.$$
$$\mathrm{sinc}(v'd_b) \approx const.$$
$$\exp(ju'p_{mn}d_b) \approx 1$$

Acoordingly, Eq.(12) can be rewritten as:

$$h'(u',\ v') \approx d_b^2\sum_m \sum_n b_{mn}\exp[jd_b\{(mu'+v'n)+u_o(m+p_{mn})\}] \tag{13}$$

By leaving the above expressions of approximation for later discussion, here we calculate conditions for the amplitude distribution on the image plane obtained through the binary filter, viz. $h'(u', v')$ of Eq.(13), to agree with that desired beforehand, viz. $g'(u', v')$ of Eq.(9). That is, Eq.(13) and (9) are necessary to be in the following relationship:

$$h'(u', v') = const.\ g'(u', v') \tag{14}$$

In the binary filter, two parameters, b_{mn} and p_{mn}, must be determined per single plane element. Therefore, respective Fourier terms of the both formulae are compared with respect to (m, n) as follows:

$$d_b^2 b_{mn} \exp\{ju_o d_b(m+p_{mn})\} = const.\ a_{mn}e^{j\varphi_{mn}/2\pi} \tag{15}$$

resulting in

$$b_{mn} = a_{mn}, \quad p_{mn}+m = \varphi_{mn}/2\pi u_o d_b \tag{16}$$

By the way, when an optical system is set up so that plane waves incident to the binary filter have the phase difference of just $2\pi N$ (N is an integer) about the x axis at the both ends of a plane element, $(2\pi N)\cdot m$ becomes independent of relative change in the phase. Accordingly, we determine as $u_o d_b = N$. At that time, b_{mn} and p_{mn} become

$$b_{mn}=a_{mn}, \quad p_{mn}= \varphi_{mn}/2\pi N \tag{17}$$

being solely determined. This has an significant meaning when realizing a binary filter, making it comprehensible that the amplitude component a_{mn} on each element at the diffraction plane should be its breadth b_{mn} and its phase component φ_{mn} be taken as $2\pi N$ times p_{mn}, the shift from the cell center to element center. In order to design so as the cell center in each element on the diffraction plane not to shift its position to hit a neighbor cell at its edge, it had better to determine as $N=1$ for higher degree of freedom.

To realize the differential filter $S(x, y) = sx$ necessary for the present study by utilizing the principle mentioned above, filter of $s|x|$ is enough to realize $\varphi_{mn} = 0$ for $x \geq 0$ and $\varphi_{mn} = \pi$ for $x < 0$, so that it is sufficient so as to satisfy:

$$\left.\begin{array}{lll} b_{mn} = s|x_{mn}| & & \\ p_{mn} = 0, & \varphi_{mn} = 0 & x \geq 0 \\ p_{mn} = 1/2, & \varphi_{mn} = \pi & x < 0 \end{array}\right\} \tag{18}$$

provided that n is constant. Accordingly the differential filter can be realized by shifting the phase by π on left and right sides.

Incidentally, the following approximation formulae were premised to realize this binary filter:

$$\left.\begin{array}{l}\mathrm{sinc}\{b_{mn}d_b(u'+u_o)\} \approx const. \\ \mathrm{sinc}(v'd_b) \approx const. \\ \exp(ju'p_{mn}d_b) \approx 1\end{array}\right\} \quad (19)$$

That is, these must hold true.

3. Simulation of a Simple Case and Its Application

First of all, we consider about $\mathrm{sinc}\{b_{mn}d_b(u'+u_o)\} \approx const.$ From $u_o d_b = N = 1$, behavior of $\mathrm{sinc}\{b_{mn}(1+du')\}$ eventually become a problem. Because not only the sinc function slowly varies, but also the range of u' is $|u'| \leq l'/2$ as mentioned before and $d_b = 1/l'$ from Eq. (6), it is found sufficient to verify $\mathrm{sinc}\{b_{mn}(1 \pm 1/2) \approx const.$. Therefore, if $b_{mn}=0.4$ for instance, the ratio of contrast at the center of the image vs. that of the both ends becomes

$$|\{\sin(0.4\pi)/0.4\pi\} - \{\sin(0.2\pi)/0.2\pi\}| \,/\, |\{\sin(0.4\pi)/0.4\pi\} + \{\sin(0.2\pi)/0.2\}| \fallingdotseq 0.6$$

and the contrast difference to this extent is no problem when judging whether this object is white or black. Because similar discussion can apply to $\mathrm{sinc}(v'd_b)$, too, we consider $\exp(ju'p_{mn}d_b)$ next. Of course, the term of exp approaches 0 near the center of $u'=0$, but p_{mn} takes a small value also at both ends of the image, so we can regard as $\exp(jp_{mn}/2) \approx 1$. Because of the above, it is nearly possible to design the differential filter by means of the binary filter.

Besides, Eq.(18) was a functional formula of the binary filter having function of the first differential operation, whereas in order to realize that having function of the second differential operation, we require the following functional formula[6]:

$$b_{mn} = s\{x_{mn}\}^2, \quad p_{mn} = 0 \qquad (20)$$

Evaluation and Conclusion

Figure 4 (a) and (b) show the transmittance distribution "F" of an object and its second differential image, respectively. As will be evident in comparison between this result and the first differential image shown in Figure 5, it is found that two sharp edges are located in parallel in the second differential image. That is, we can regard that the second differentiation was performed.

Here, Figure 5 is the result (output) that was obtained when an actual photographic data (input) was passed through an experimentally fabricated differentiation filter in the u, v direction simultaneously, indicating that the double differentiation was effectively executed. Accordingly, it was made clear that the digital processing could also be filtered in two dimensions.

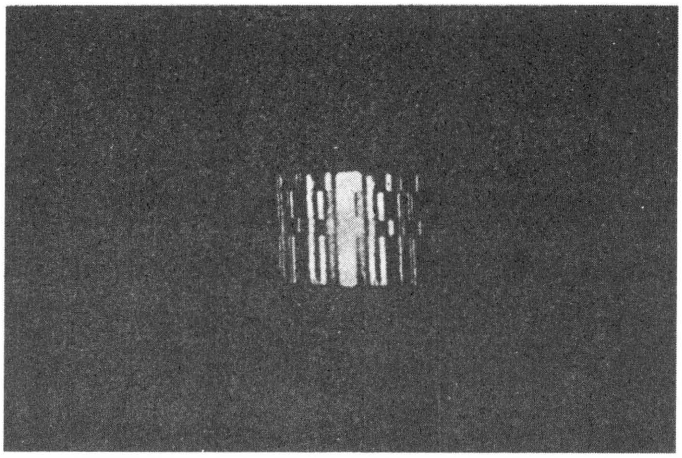

Figure 4 Object with a letter of "F" image (center) and optically secondary differentiated images (both sides)

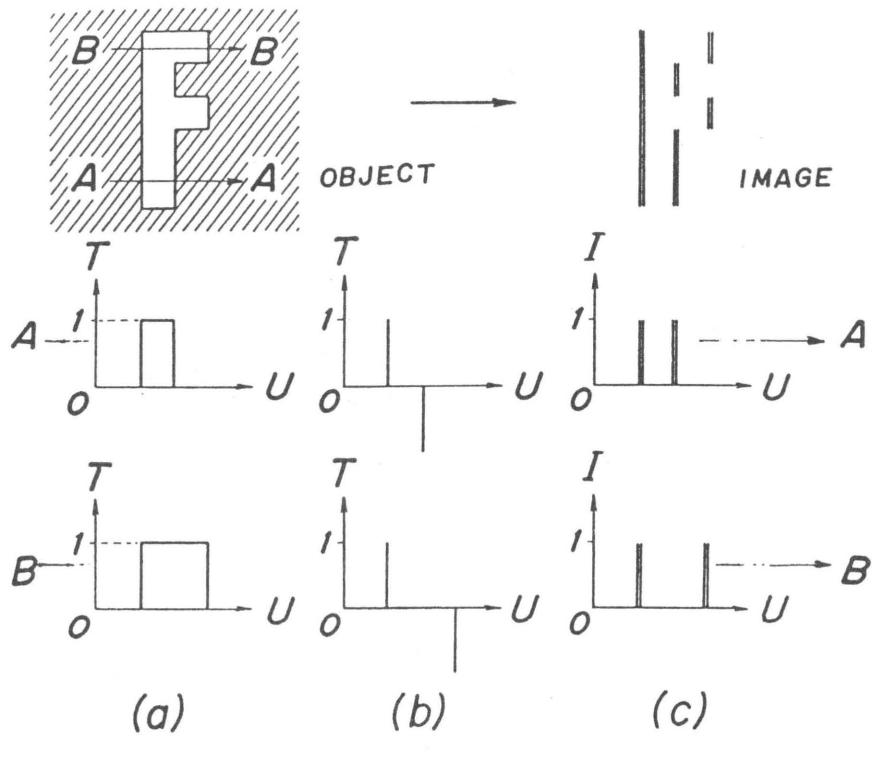

Figure 5 Principle of image differnciations
(a) Object (b) first differenciation (c) second differenciation

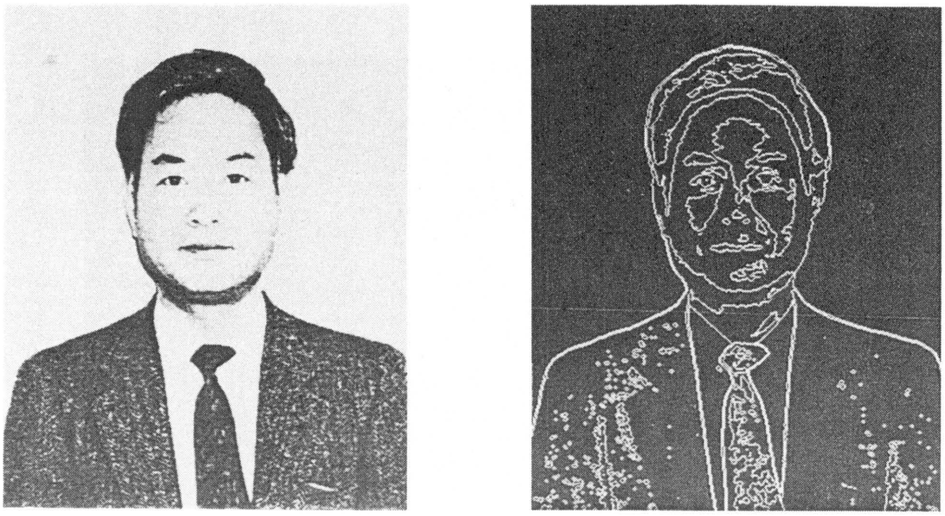

Figure 6 Application of general image (left) and secondary differentiated image (right)

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和文抄録

微分像用バイナリーフィルターの設計と製作（その１）

長江貞彦

本研究は、来るべき Opto-computing 時代の到来を見込んで、あえて analog 的な光学系に digital 的なフィルタを設計し、画像に微分演算を行うことを試みたものである。
一般に、物体を単色光で照明し、その像を作るとき、その結像光学系を再回折光学系という。本研究における理論および実験はすべて近軸光線の条件を満たす再回折光学系によるものとし、光源はレーザー光のような coherent 光源を用いるものとする。

さて、結像系を情報通信理論の立場から考えると、その取扱いが簡単となり、種々の問題に対して有効な働きを示すことが多い。いま、coherent に照明された結像系において、通信系における時間軸を光学系の空間軸に対比させ、物体を透過した波面の振幅分布を入力、像面における波面の振幅分布を出力と考えると、光学系を一種の線形伝送系とみなすことができる。そこで、この伝送系の特性を積極的に変えるため、空間フィルタ（spatial filter）を光学系のスペクトル面、すなわち物体直後のレンズの後側焦点面の Fraunfoher 回折面に挿入すると、物体と像面との振幅透過率分布の間に、種々なる数学的演算が可能となる。

本研究ではこの演算の一つとして、光学的微分演算の原理と、それに必要な光学的微分 filter の設計と製作を行なった。また、簡単な実験により、その性能評価について考察した。その結果、原理的には、バイナリーフィルタが有効であり、今後オプト・コンピューティングにも十分な活用が可能であることがわかった。