Theoretical Formula of Underwater Visibility

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Introduction

The underwater visibility of object is determined for the synthesic action of the physical factors and of the visual physiology,but hitherto the study of the underwater visibilty has been only made the experimental formula from the measured values. And there are few researches from the basis of the optical theory and the threshold of human eye.

The author measured the underwater visibility using the diffused flourescent light which illuminate to the tank water from the upper position and alterring the physical factors. namely, net construction,diameter,colour,reflectance,transparent,turbidity and illumination of tank water.

In this paper,the theoretical formula of the visibility in meteorology was applied to more expermiental condition and their theoretical analysis was discussed by using the experimental values.

Discussion

The visibility of the object is dependent on the luminance, Bo of the water space in the solid angle which the object subtend to the eye and on the background luminance,Bh, The contrast,C is defined by the equation,

$$C = \left| \frac{Bo - Bh}{Bh} \right| \qquad (1)$$

where Bo and Bh can be formulated in accordance with Middleton[1] as follow. Consider an volume element,dV

$$dV = x^2 . d\omega . dx \qquad (2)$$

of the cone of water (Fig. 1) the base of which is a portion of object and the apex of which is at the observer's eye.

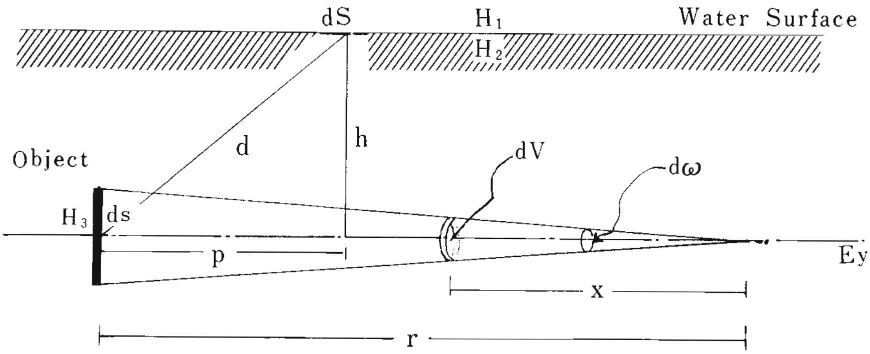

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Fig. 1 Schematic diagram for estimating the brightness contrast

The volume element, dV is assumed to be illuminated to the same extent and in the same way no matter what the value of x. The intensity, dI_1, of the volume element in the direction of eye will be

$$dI_1 = dV.\beta\,(90^\circ).E \tag{3}$$

where E is the horizontal illuminance at the volume element and $\beta\,(90^\circ)$ is the volume scattering function. As $\beta\,(90^\circ)=A\cdot K$ is assumed,

$$dI_1 = dV\cdot A\cdot K\cdot E \tag{3$'$}$$

where A is a constant proportionality, to be determined from boundary conditions. Here an assumption is also introduced that the volume scattering function, $\beta\,(90^\circ)$ is approximately to the attenuation coefficient, K.

The illuminance, dE at the eye due to this light scattered from dV is

$$dE = dI_1\cdot x^{-2}\cdot e^{-Kx} \tag{4}$$

The radiant emittance, dB at the eye due to the volume element is

$$dB = dE\cdot d\omega^{-1} \tag{5}$$

where $d\omega$ is a solid angle which dV subtends at the eye. From the equations, (2), (3)$'$, (4) and (5),

$$dB = A\cdot K\cdot E\cdot e^{-Kx}dx \tag{6}$$

Now integrating the equation (6) from $x=0$ to $x=r$,

$$Br = \int_{x=0}^{x=r} A\cdot K\cdot E\cdot e^{-Kx}dx = AE\,(1-e^{-Kr}) \tag{7}$$

where r is a visual range of the object and Br is a total luminance of the water in the cone of a solid angle. Similarly, integrating the equation, (6) from $x=0$ to $x=L$ which is the length of water tank, the luminanceoof the background, B_L is

$$B_L = A\cdot E\,(1-e^{-KL}) \tag{8}$$

where B_L is a total luminance of background. But, inner side of tank is painted by black enamel, the reflectance from it negligibly reduced. Therefore, the background luminance, Bh of the equation, (1) can be substitude for B_L.

All the objects produced the luminance by the scattered light in the water tank and by the straight light from the horizontal water surface. Here the former is B_T and the latter is Bd. The luminance, B_T produced by the scattered light at the observer's eye will be

$$B_T = R\cdot Br\cdot e^{-Kr} \tag{9}$$

where R is the reflectance of objects.

And Bd can be obtained as follows. In Fig. 1, the radiant emittance to a part of under water surface, dS and the radiant intensity, dI_1 in the direction of the object will be

$$dI_1 = dS \cdot \cos\theta \cdot dBs \tag{10}$$

Here, assumed that dS is the perfect diffuser

$$dBs = \frac{H_2}{\pi} \tag{11}$$

where H_2 is the downward irradiance just below the surface. The illuminance, H_3 in the point P of the target which is illuminated from the part of dS

$$H_3 = \frac{dI \cdot \cos\left(\frac{\pi}{2} - \theta\right)}{(h\sec\theta)^2} \cdot e^{-Kh\sec\theta} \tag{12}$$

If it is assumed that the object is the perfect diffuser and the radiant emittance, dBp produced by H_3 in the direction of the observer's eye will be

$$dBp = R \cdot \frac{H_3}{\pi} \tag{13}$$

The luminance, dBd at the observer's eye due to this radiant emittance is

$$dBd = dBp \cdot e^{-Kr} = \frac{R \cdot \sin\theta \cdot dI_1}{\pi\,(h\sec\theta)^2} \cdot e^{-K(h\sec\theta + r)} \tag{14}$$

From the equations (10), (11), (12), (13) and (14),

$$dBd = \frac{R \cdot H_2 \cdot \sin\theta \cdot \cos^3\theta}{(\pi r)^2} \cdot e^{-K(h\sec\theta + r)} dS \tag{15}$$

In Fig. 1, if PY is the length,

$$\sin\theta = \frac{p}{\sqrt{h^2+p^2}}, \qquad \cos\theta = \frac{h}{\sqrt{h^2+p^2}}$$

Substituting these into the equation (15),

$$dBd = \frac{R \cdot H_2 \cdot h}{\pi^2} \cdot \frac{p}{(h^2+p^2)^2}\, e^{-K(\sqrt{h^2+p^2}+r)} dS \tag{16}$$

We must integrate the eqution (16) in the forward all surface square of the target and as the widths of water tank was narraw, the author assumed $dS = dP \cdot Q$ not integrating it aproximately. And the equation (16) takes the form

$$dBd = \frac{R \cdot H_2 \cdot Q \cdot h}{\pi^2} \cdot \frac{p}{(h^2+p^2)^2} \cdot e^{-K(\sqrt{h^2+p^2}+r} dp \tag{17}$$

where Q is the width of water tank.

Now, integrating the equation, (17) from $p=0$ to $p=r$,

$$Bd=\frac{R\cdot H_2\cdot Q\cdot h}{\pi^2}\cdot\int_{p=0}^{p=r}\frac{p}{(h^2+p^2)^2}\cdot e^{-K(\sqrt{h^2+p^2}+r)}dp \tag{18}$$

The luminance, Bo produced by the point P of the object in the direction of observer's eye is

$$Bo=Br+B_T+Bd \tag{19}$$

Namely, the contrast, C is

$$C=\left|\frac{Bo-B_L}{B_L}\right|=\left|\frac{Br+B_T+Bd-B_L}{B_L}\right| \tag{20}$$

From the equations, (7), (9), (18) and (8), the equation, (20) will be

$$\varepsilon=\left|\frac{1}{A\cdot E\cdot(1-e^{-KL})}\cdot\frac{R\cdot H_2\cdot h\cdot Q}{\pi^2}\cdot\int_{p=0}^{p=r}e^{-K(\sqrt{h^2+p^2}+r)}dp+\frac{R\cdot e^{-Kr}(1-e^{-Kr})+e^{-KL}-e^{-Kr}}{1-e^{-KL}}\right| \tag{21}$$

2-i) The determination of A

As above mentioned, R is measured by the integrating sphere and Q is the width of water. The illuminance, H_2 just below the water surface is gained from each measured value in the water and h is the depth. Then, obtaining the value of A, the apparatus shown in **Fig. 2** is used.

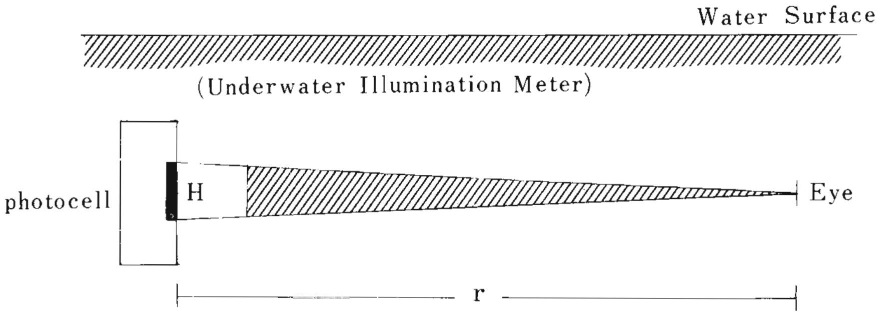

Fig. 2 Schematic diagram for estimating the illuminance, H.

Inspecting the direct light from the water surface, the photocell is set to the place of the observer's eye and the illuminance is measured.

Accordingly, the illuminance of the detector in the direction of the observer's eye is

$$H=E\cdot A\cdot K\cdot\omega\int_{x=0}^{x=r}e^{-Kx}dx=E\cdot A\cdot\omega\,(1-e^{-Kr})\qquad(22)$$

where θ can obtain geometrically, whose is a plain angle. Namely,

$$\omega=2\pi\,(1-\cos\theta)$$

2-ii) The examination of the equation (21)

Substituting the measured values, R, Q, H_2, h and r in the equation, (21), the threshold of brightness contrast can be computed. The relation between the threshold of brightness contrast and the visual angle will be linear as Blackwell's[2] result.

Referrence

1) Middleton, W. E. K. Univ. Toront Press, Toront, Ont., 250pp.
2) Blackwell, H. R. J. Opt. Soc. Amer., 36, 624-643(1946)

水中視程の理論式について

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一般に、水中での物体の視程は物理的要因と視覚生理との総合作用の結果として決まるものであるが、従来の水中視程の研究は主として人間の目による測定値から実験式を出す程度にとどまって､光学理論及び人間の目の識閾の法則を基とした解析的研究は少なかった。

著者は前報でのべた様にKoschmieder, Middletonの大気中の視程に関する理論式をさらに実験条件に合う様に書きかえた。

今回の論文は実験よりも、理論式の立てかたに重点を置いた為、実験はおこなっていないが、この理論式を使って出した識閾の値と視角との関係は前報の結果とほぼ一致するものと思われる。