平成３０年度
「人・環境・エネルギーの未来創造～“生命による情報利用”からのアプローチ～」
「エントロピーを鍵とする「情報・生命・エネルギー」の包括的な理解」に関する
研究報告

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共同研究者　前田健吾（芝浦工業大学）
Eric Mefford（カリフォルニア大学サンタバーバラ校）

２．研究題目：「量子論的光的エネルギー条件の破れとワームホール時空」

３．概要：　エントロピーと情報に関する幾何学的および量子論的考察として、量子論的光的エネルギー条件「Quantum Null Energy Condition (QNEC)」を、AdS/CFT 対応（ホログラフィック理論）を用いて吟味した。

一般に、物理学における理論モデルは安定な基底状態をもつべきであり、そのためには理論の定めるエネルギーに下限がなければならない。例えば、一般相対論においては、エネルギー・運動量テンソルに対する制限が課される。代表的なものは、弱いエネルギー条件「Weak Energy Condition (WEC)」であり、「どのような観測者からもエネルギー密度が非負であれ」という主張である。WEC は既知の古典的な物質場に対しては必ず成り立つものと考えられている。WEC の光的極限に対応するものは光的エネルギー条件「Null Energy Condition (NEC)」である。NEC は、特異点定理、トポロジー定理など、重力理論のまつわる諸定理の証明において鍵となる重要な役割を果たす。一方、物質を量子論的に扱う場合には、カシミア効果等で良く知られているように、少なくとも局所的には必ず WEC や NEC を破ることができる。そこで、これまで様々な型の非局所的エネルギー条件が提案されてきた。最近では、量子論的光的エネルギー条件(QNEC)がある。これは余次元２の閉曲面に対するベッケンシュタイン・ホーキング（Bekenstein-Hawking）エントロピーと量子場のフォン・ノイマン・エントロピーを結びつけた「一般化されたエントロピー」に対する熱力学第 2 法則と直結するものである。これまでに平坦な時空上で QNEC がさまざまな量子場に対して成立することが証明されてきたが、曲がった時空上での量子場に対しては QNEC はほとんど吟味されてこなかった。

本研究では、QNEC を曲がった時空上で吟味するために、背景時空として 3 次元の漸近平坦ワームホール時空を考え、それを境界にもつ 4 次元の漸近 AdS ブラックホール解を構成した。そして、AdS/CFT 対応を用いて強結合量子場に対して QNEC を評価すると QNEC が破れることを示した。有限温度のブラックホールの双対理論として、QNEC に対して有限温度の場の量子論の効果を考えた。このモデルでは、特にフォン・ノイマン・エントロピーに、紫外発散に加えて赤外発散も現

れるため、赤外正則化の一つの方法として、対応するバルク・ブラックホールのベッケンシュタイン・ホーキングエントロピーによって赤外発散を相殺することを提案した。この方法は有限温度の効果とエンタングルメント・エントロピーの部分を分離可能とする観点で、自然な正則化と考えられるのが特徴である。

４．研究成果発表：学術誌 Physical Review D に以下のように掲載された。
“Violation of the quantum null-energy condition in a holographic wormhole and infrared effects” Physical Review D99 (2019) 026004

また、国内の研究機関において講演を行った（東京大学 2018 年 10 月 29 日、京都大学 2018 年 10 月 31 日、大阪大学 2019 年 1 月 8 日、名古屋大学 2019 年 1 月 29 日）。以下にこれらの講演で用いた発表ファイルを載せることで、本研究のより詳しい内容紹介の代わりとしたい。

（次項に続く）

Violation of the Quantum Null Energy Condition in a holographic wormhole

Akihiro Ishibashi
(Kindai U.)

Based on PRD99, 026004 (2019)

joint work w/ Kengo Maeda (Shibaura I. T.)
Eric Mefford (UCSB)

1

Introduction

"Energy" should be bounded below, for a physically sensible system to have a stable ground state.

It is well-known that quantum field effects violate classical, local positive energy conditions.

Quantum Null Energy Condition (QNEC) is a conjectured lower bound on $\langle T_{ab} \rangle$ in any quantum state.

We test QNEC in a wormhole spacetime by using holographic methods and find the violation of QNEC.

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Outline

- (I) Null Energy Condition (NEC)
- (II) Quantum Focusing Conjecture
  and Quantum NEC (QNEC)
- (III) Constructing AdS-BH w. wormhole boundary
- (IV) Testing QNEC on the wormhole spacetime

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Energy Conditions

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Physically sensible conditions for Stress-Energy tensor

RHS of the Einstein equations $R_{ab} - \frac{1}{2} g_{ab} R = 8\pi G T_{ab}$

Example: FLRW cosmology $T_{ab} = \rho u_a u_b + P h_{ab}$

Weak Energy Conditions (WEC)

For any timelike vector ξ^a ➡ $T_{ab}\xi^a\xi^b \geqslant 0$

- holds for classical matter system Ex. FLRW case $\rho \geqslant 0 \quad \rho + P \geqslant 0$

Dominant Energy Conditions (DEC)

For any future dir. timelike vector ξ^a

$-T^a{}_b\xi^b$ also is a future dir. timelike or null vector

- Conservation theorem Ex. FLRW case $\rho \geqslant |P|$

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Focusing conditions for geodesic congruences

Strong Energy Conditions (SEC)

For any timelike vector ξ^a ➡ $\left(T_{ab} - \frac{1}{2} T g_{ab}\right)\xi^a\xi^b \geqslant 0$

- Timelike Focusing $R_{ab}\xi^a\xi^b \geqslant 0$

Ex. FLRW case $\rho + 3P \geqslant 0 \quad \rho + P \geqslant 0$

Null Energy Conditions (NEC)

For any null vector k^a ➡ $T_{ab}k^a k^b \geqslant 0$

- Null Focusing $R_{ab}k^a k^b \geqslant 0$ Null-limit of WEC

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NEC, QFC and QNEC

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Null Energy Condition

Null Energy Condition (NEC) plays an important role in GR

$$T_{kk} := T_{ab}k^a k^b \geqslant 0$$

- holds for classical matter systems

It governs the focusing of null geodesic congruence, from which one can learn a lot about the spacetime geometry under consideration.

It is used in an essential way in the proof of

Singularity theorems
Area theorem
Topology censorship etc.

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Classical Focusing theorem

Surface orthogonal null

$$k^a = \left(\frac{d}{d\lambda}\right)^a$$

Classical expansion

$$\theta = \nabla_a k^a = \frac{1}{\mathcal{A}}\frac{d\mathcal{A}}{d\lambda}$$

Shear

$$\sigma_{ab}$$

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Classical Focusing theorem

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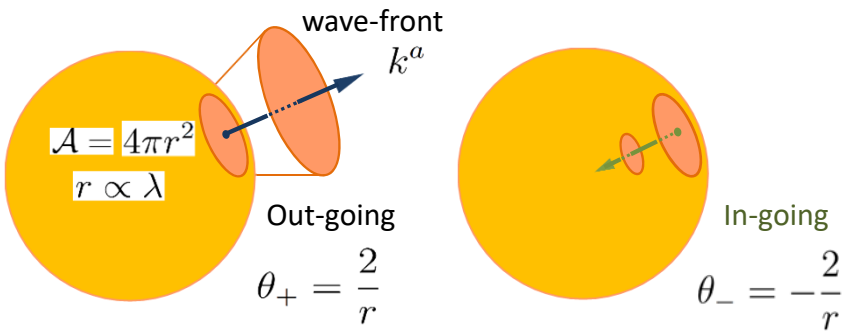
Shear

$$\sigma_{ab}$$

Ex: light-rays emanating from 2-sphere in flat spacetime



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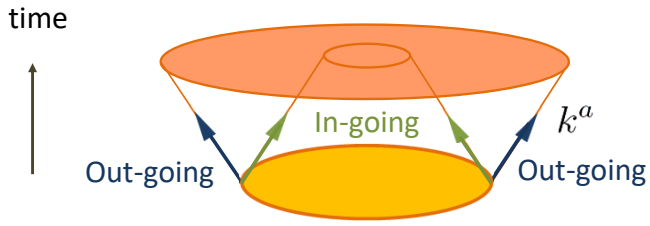

$$\theta_+ = \frac{2}{r} > 0$$

$$\theta_- = -\frac{2}{r} < 0$$

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Classical Focusing theorem

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Classical expansion

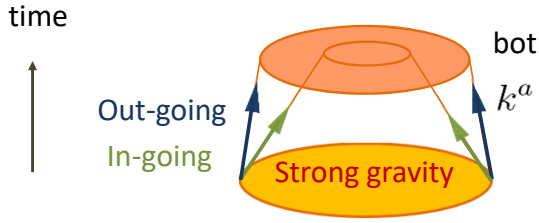
$$\theta = \nabla_a k^a = \frac{1}{\mathcal{A}}\frac{d\mathcal{A}}{d\lambda}$$

Shear

$$\sigma_{ab}$$

Ex: light-rays emanating from 2-sphere in curved spacetime

e.g. inside BH



both (out-, in-going) expansions can be negative

$$\theta_+ < 0 \qquad \theta_- < 0$$

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Classical Focusing theorem

Surface orthogonal null

$$k^a = \left(\frac{d}{d\lambda}\right)^a$$

Classical expansion

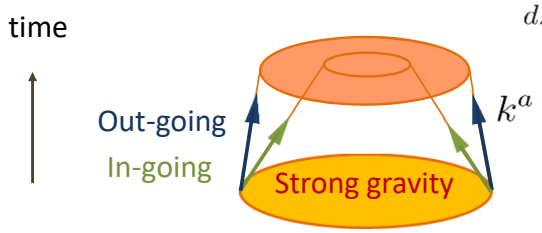
$$\theta = \nabla_a k^a = \frac{1}{\mathcal{A}}\frac{d\mathcal{A}}{d\lambda}$$

Shear

$$\sigma_{ab}$$

In curved spacetime, the expansion obeys Raychaudhuri equation

$$\frac{d\theta}{d\lambda} = -\frac{1}{D-2}\theta^2 - \sigma^2 - T_{kk}$$



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Classical Focusing theorem

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Classical expansion

$$\theta = \nabla_a k^a = \frac{1}{\mathcal{A}}\frac{d\mathcal{A}}{d\lambda}$$

Shear

$$\sigma_{ab}$$

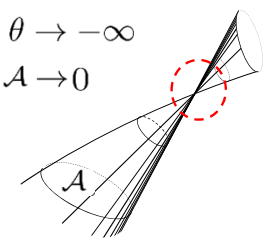
Raychaudhuri equation

$$\frac{d\theta}{d\lambda} = -\frac{1}{D-2}\theta^2 - \sigma^2 - T_{kk}$$

$T_{kk} \geqslant 0$ ➡ $\boxed{\frac{d\theta}{d\lambda} \leqslant 0}$ ➡ if initially $\theta_0 < 0$

$\theta \to -\infty$

$\mathcal{A} \to 0$



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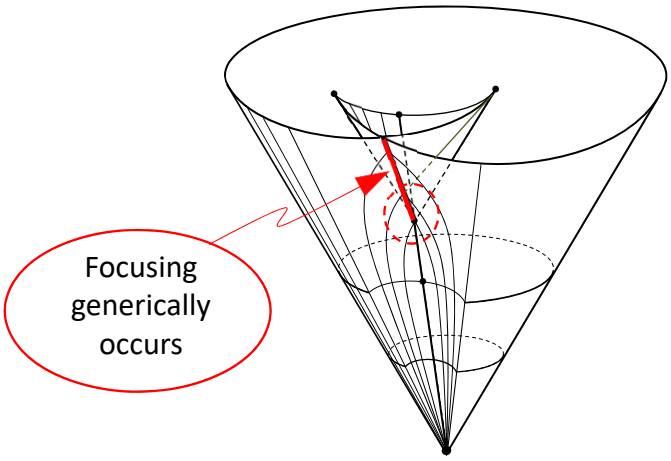
Classical Focusing theorem

Focusing $\theta \to -\infty$ of null geodesic congruence generically occurs under NEC and Null generic condition/initial convergence $\theta_0 < 0$



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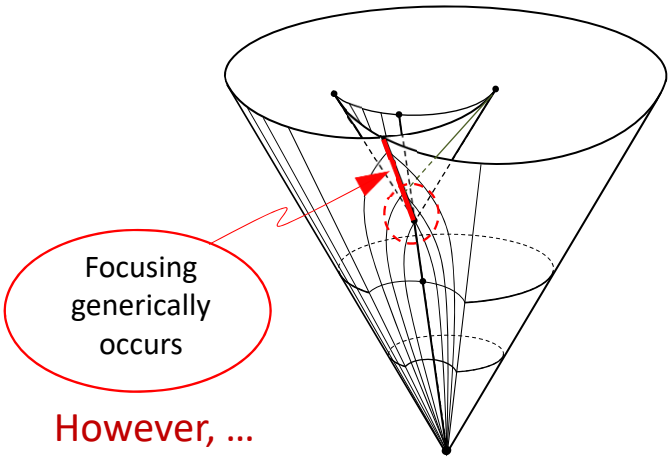

However, …

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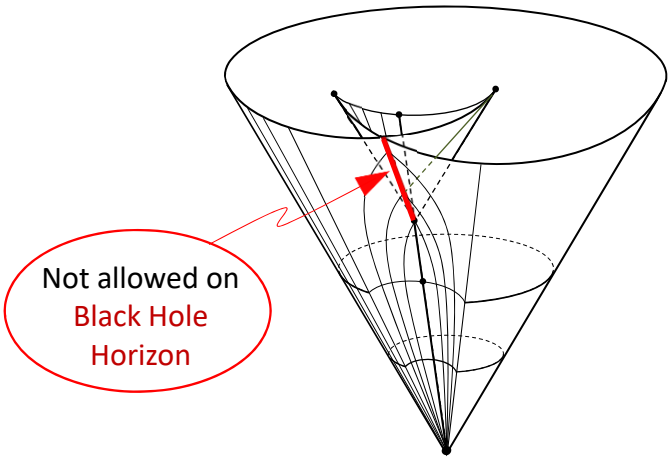
Area theorem and BH 2nd Law

Causal nature of the BH event horizon does not allow focusing $\theta \to -\infty$ of the null generators toward future.



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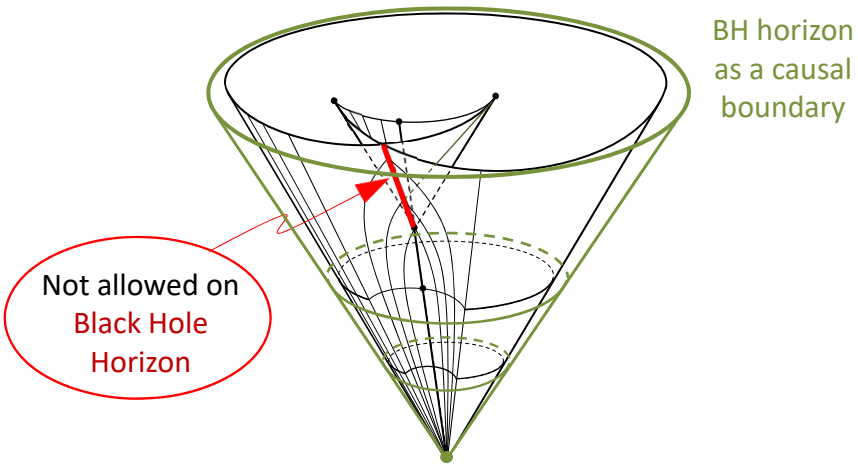

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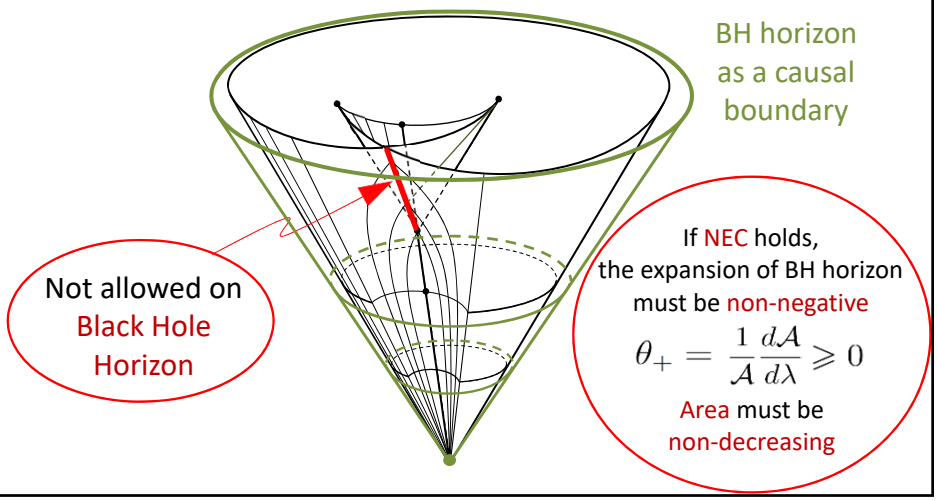

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Area theorem and BH 2nd Law

Causal nature of the BH event horizon does not allow focusing $\theta \to -\infty$ of the null generators toward future.

Classical Focusing w. NEC → $\boxed{\delta\mathcal{A} \geqslant 0}$ Hawking 71

BH must have its own Entropy $S_{BH} = \frac{\mathcal{A}}{4G\hbar}$ $\delta S_{BH} \geqslant 0$ Bekenstein 72

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Violation of NEC and Non-local conditions

NEC (any local Energy Conditions) can be violated by Quantum Fields.

$$\cancel{T_{kk} \geqslant 0} \qquad \langle T_{ab} \rangle k^a \cancel{k^b \geqslant} 0$$

- by e.g. Hawking-radiation, Casimir effects

Averaged Null Energy Conditions (ANEC)

$$\int \langle T_{ab} \rangle k^a k^b d\lambda \geqslant 0$$

Non-local: defined along any complete null geodesic

e.g. Wald-Yurtsever 91

ANEC can be violated by Quantum Fields.

$$\int \langle T_{ab} \rangle k^a k^b \cancel{d\lambda \geqslant} 0$$

e.g. Visser 96

Achronal averaged Null Energy Conditions (AANEC)

e.g. Graham and Olum 07

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Area theorem and BH 2nd Law

Causal nature of the BH event horizon does not allow focusing $\theta \to -\infty$ of the null generators toward future.

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Generalized Entropy $S_{gen} = S_{BH} + S_{out}$ Bekenstein 73

$S_{out} = -\mathrm{Tr}\rho_{out} \log \rho_{out}$
von Neumann entropy

Generalized 2nd Law $\boxed{\delta S_{gen} \geqslant 0}$ e.g. Page 76, Unruh-Wald 82, Zurek-Thorne 85, Frolov-Page 93

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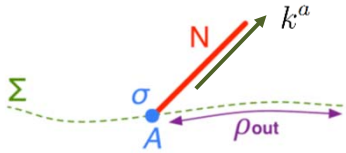
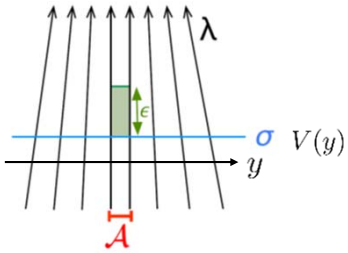
Quantum Focusing Conjecture

Bousso-Fisher-Leichenauer-Wall 16

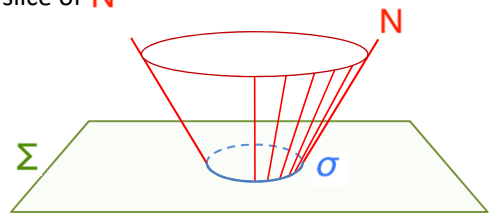
Generalized entropy for any co-dim. 2 σ splitting Cauchy surface Σ





N : Null hypersurface orthogonal to σ

$V(y)$ defines a slice of N



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Bousso-Fisher-Leichenauer-Wall 16

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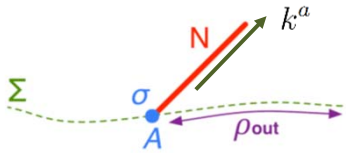

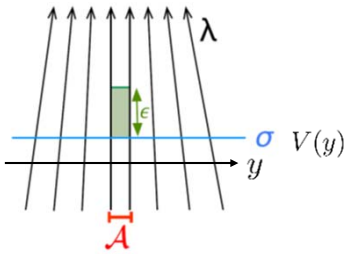

N : Null hypersurface orthogonal to σ

$V(y)$ defines a slice of N

Generalized entropy for σ

$$S_{gen}[V(y)] = \frac{\mathcal{A}[V(y)]}{4G\hbar} + S_{out}$$

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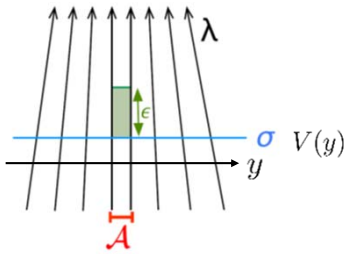
Quantum Focusing Conjecture

Bousso-Fisher-Leichenauer-Wall 16

Quantum expansion Θ as functional derivative of S_{gen}

$V(y)$ defines a slice of N

$V_\epsilon(y) := V(y) + \epsilon\delta(y - y_1)$



$$\Theta[V(y); y_1] := \lim_{\mathcal{A}\to 0} \frac{4G\hbar}{\mathcal{A}} \frac{dS_{gen}}{d\epsilon} = \frac{4G\hbar}{\sqrt{{}^V g(y_1)}} \frac{\delta S_{gen}}{\delta V(y_1)}$$

Quantum Focusing Conjecture

$$\frac{\delta}{\delta V(y_2)} \Theta[V(y); y_1] \leqslant 0$$

c.f. Classical Focusing

$$\frac{d\theta}{d\lambda} \leqslant 0$$

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Quantum Focusing Conjecture

Bousso-Fisher-Leichenauer-Wall 16

Quantum expansion Θ as functional derivative of S_{gen}

QFC

$$\frac{\delta}{\delta V(y_2)} \Theta[V(y); y_1] \leqslant 0$$

Off-diagonal part of QFC $y_2 \neq y_1$

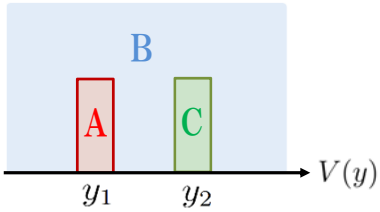
$$\frac{\delta^2 S_{gen}}{\delta V(y_2)\, \delta V(y_1)} \leqslant 0$$

$$\propto \lim_{\epsilon_1, \epsilon_2 \to 0} \frac{1}{\epsilon_1 \epsilon_2} \left\{ S_{out}[V_{1,2}] - S_{out}[V_1] - S_{out}[V_2] + S_{out}[V] \right\} \leqslant 0$$

This follows from strong subadditivity

$$S_{out}[V] + S_{out}[V_{1,2}] \leqslant S_{out}[V_2] + S_{out}[V_1]$$

$$S(ABC) + S(B) \leqslant S(AB) + S(BC)$$



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Quantum Focusing Conjecture

Bousso-Fisher-Leichenauer-Wall 16

Quantum expansion Θ as functional derivative of S_{gen}

QFC

$$\frac{\delta}{\delta V(y_2)}\Theta[V(y);y_1] \leqslant 0$$

Diagonal part of QFC $y_2 = y_1$

$$\Theta' := \frac{d\Theta}{d\lambda} \leqslant 0$$

Classical Focusing to Quantum Focusing $\theta \longrightarrow \Theta := \theta + \frac{4G\hbar}{\mathcal{A}} S'_{out}$

$$\theta' \leqslant 0 \quad \Rightarrow \quad \Theta' \leqslant 0$$

Classical Focusing Theorem holds under NEC

Quantum Focusing Conjecture believed to hold irrespective of NEC

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Quantum Null Energy Conditions (QNEC)

Bousso-Fisher-Leichenauer-Wall 16
Bousso-Fisher-Koeller-Leichenauer-Wall 16

QFC to Quantum Null Energy Conditions $\Theta := \theta + \frac{4G\hbar}{\mathcal{A}} S'_{out}$

$$0 \geqslant \Theta' = \underline{\theta'} + \frac{4G\hbar}{\mathcal{A}}(S''_{out} - \theta S'_{out})$$

$$= \underline{-\frac{1}{D-2}\theta^2 - \sigma^2 - 8\pi G \langle T_{kk} \rangle} + \frac{4G\hbar}{\mathcal{A}}(S''_{out} - \theta S'_{out})$$

On σ w/ vanishing classical expansion and shear $\theta = 0 \quad \sigma_{ab} = 0$

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Quantum Null Energy Conditions (QNEC)

Bousso-Fisher-Leichenauer-Wall 16
Bousso-Fisher-Koeller-Leichenauer-Wall 16

QFC to Quantum Null Energy Conditions $\Theta := \theta + \frac{4G\hbar}{\mathcal{A}} S'_{out}$

$$0 \geqslant \Theta' = \theta' + \frac{4G\hbar}{\mathcal{A}}(S''_{out} - \theta S'_{out})$$
$$= -\frac{1}{D-2}\cancel{\theta^2} - \cancel{\sigma^2} - 8\pi G \langle T_{kk} \rangle + \frac{4G\hbar}{\mathcal{A}}(S''_{out} - \cancel{\theta S'_{out}})$$

On σ w/ vanishing classical expansion and shear $\theta = 0 \quad \sigma_{ab} = 0$

$\Rightarrow$ $\boxed{\text{QNEC} \quad \langle T_{kk} \rangle \geqslant \frac{\hbar}{2\pi\mathcal{A}} S''_{out}}$

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Quantum Null Energy Conditions (QNEC)

Bousso-Fisher-Leichenauer-Wall 16
Bousso-Fisher-Koeller-Leichenauer-Wall 16

$$\boxed{\text{QNEC} \quad \langle T_{kk} \rangle \geqslant \frac{\hbar}{2\pi\mathcal{A}} S''_{out}}$$

Purely Quantum statements since G drops out as an overall factor

$\Rightarrow$ Proof within Quantum Field Theory in Minkowski background

e.g. For free fields Bousso-Fisher-Koeller-Leichenauer-Wall 16

e.g. For general interacting theories
Balakrishnan-Faulkner-Khandker-Wang 17

c.f. holographic proof with Gravity dual for interacting theories
Koeller-Leichenauer 16

c.f. Violation of QFC for Gauss-Bonnet theories, indicating violation of QNEC
Fu-Koeller-Marolf 16

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Curved space QNEC Fu-Koeller-Marolf 17

"local" stationarity condition

- For $d \leqslant 3$, QNEC holds under $\theta = 0$

-- consistent w/. original QNEC conjecture that required

$$\sigma_{ab}|_p = \theta|_p = \dot{\theta}|_p = 0$$ Bousso-Fisher-Leichenauer-Wall 16

e.g. at a point p on a Killing horizon

Stationarity conditions.

- For $d = 4, 5$ more conditions are required.
  (vanishing of additional derivatives and Dominant EC)

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AdS-black hole w/ boundary wormhole

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4D AdS-black hole

$$ds^2 = \frac{1}{z^2}\left[-f_k(z)dt^2 + \frac{dz^2}{f_k(z)} + d\Sigma_k^2\right]$$

$$f_k(z) = 1 + k\,z^2 - \mu\,z^3$$

$$d\Sigma_k^2 \begin{cases} k = 0 & \text{planar metric} \\ k = -1 & \text{hyperbolic metric} \end{cases}$$

3D boundary

$$k = 0 \quad ds^2_{bdry} = -dt^2 + dr^2 + r^2\,d\phi^2$$

$$k = -1 \quad ds^2_{bdry} = -dt^2 + dr^2 + \cosh^2 r\;d\phi^2$$

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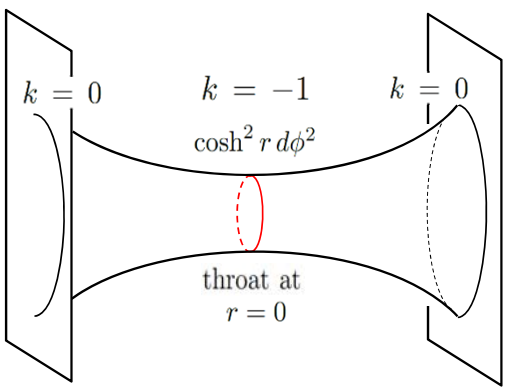
Idea



Attempt to connect a hyperbolic region around the center and 2 copies of asymptotically flat regions

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Numerical construction

Ansatz

$$ds^2 = \frac{1}{g(x)^2 y^2}\left[-(1-y)\, f(x,y) T dt^2 + \frac{g(x)^2 A}{(1-y) f(x,y)} dy^2 + \frac{4B(dx + x(1-x^2)^2 F dy)^2}{(1-x^2)^4} + \frac{\ell(x) S}{(1-x^2)^2} d\phi^2\right]$$

$$f(x,y) = 1 + y + y^2 x^2 (3 - 2x^2)$$

$X \equiv \{T, S, A, B, F\}$ -functions of x, y

Expand these functions wrt

$$x(z,r) \approx r + z^2 x^{(2)}(r) + z^4 x^{(4)}(r) + \mathcal{O}(z^5)$$

$$y(z,r) \approx z\left[\frac{1}{g(r)} + z^2 y^{(3)}(r) + z^3 y^{(4)}(r) + \mathcal{O}(z^4)\right]$$

Boundary conditions

$\underline{x=0}$: $\partial_x X = 0$

$\underline{x=1}$: $T, S, A, B = 1 \quad F = 0$

$\underline{y=0}$: $T, S, A, B = 1 \quad F = 0$

$\underline{y=1}$: $\frac{T}{A} = 1$

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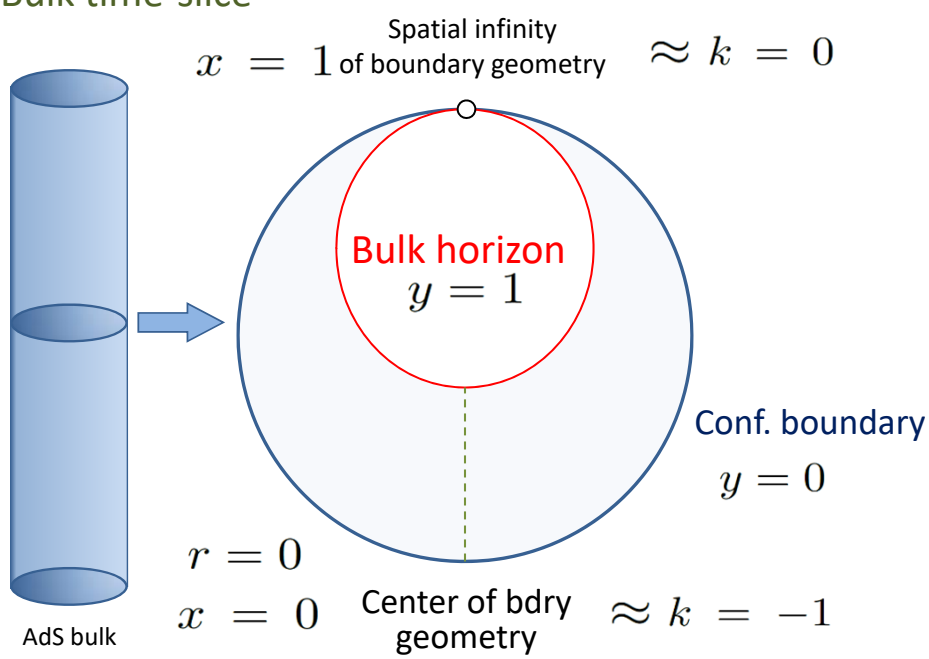
Bulk time-slice



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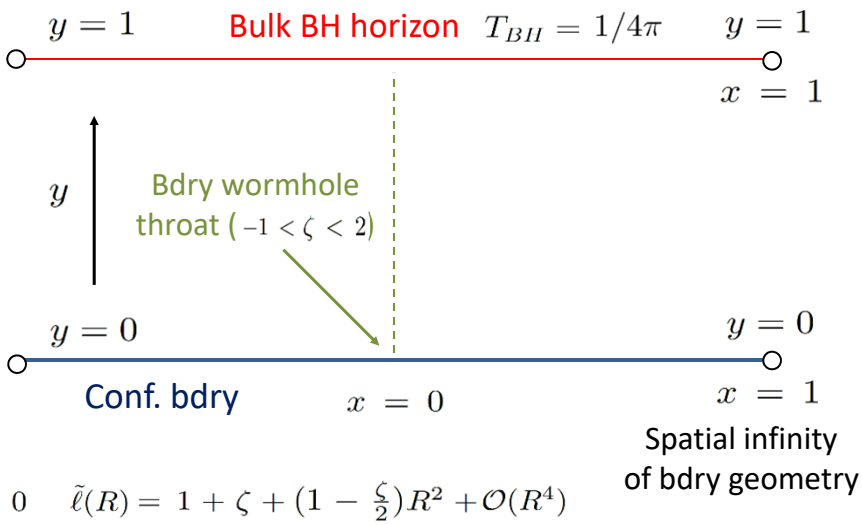
Bulk time-slice : Numerical solution



$x = 0 \quad \tilde{\ell}(R) = 1 + \zeta + (1 - \frac{\zeta}{2})R^2 + \mathcal{O}(R^4)$

$x = 1 \quad \tilde{\ell}(R) = R^2 + \mathcal{O}(R)$

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Testing QNEC

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Testing QNEC in boundary wormhole

Both side can compute by using holographic methods

$$\langle T_{kk} \rangle \geqslant \frac{\hbar}{2\pi \mathcal{A}} S''_{out}$$

LHS: As a holographic Stress-energy tensor

RHS: As a holographic Entanglement Entropy

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Testing QNEC in boundary wormhole

LHS: $\langle T_{kk} \rangle$ holographic Stress-energy tensor

Fefferman-Graham expansion

$$ds^2 = \frac{dz^2}{z^2} + \frac{1}{z^2}\left[h^{(0)}_{ab} + z^2 h^{(2)}_{ab} + z^3 h^{(3)}_{ab}\right] dx^a dx^b$$

$$\langle T_{ab} \rangle = \frac{3h^{(3)}_{ab}}{16\pi G}$$

de Haro-Solodukhin-Skenderis 01

- contract w/ null vector $k^a = \partial_t + \partial_R$
- can check the off-diagonal terms vanish and

$\langle T_{kk} \rangle < 0$ near the wormhole throat

$\langle T_{kk} \rangle > 0$ near spatial infinity

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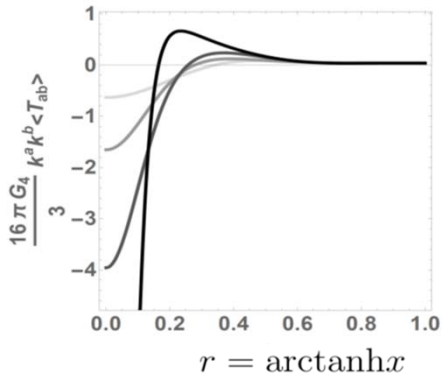
Testing QNEC in boundary wormhole

LHS: $\langle T_{kk} \rangle$ holographic Stress-energy tensor

Numerical evaluation of $\langle T_{kk} \rangle$



Classical NEC is violated near the wormhole throat

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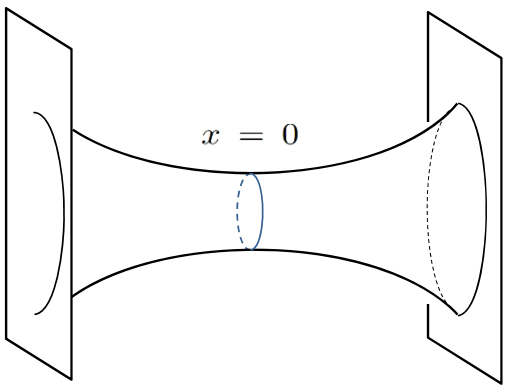
Testing QNEC in boundary wormhole

RHS: S_{out} Half-space Entanglement Entropy



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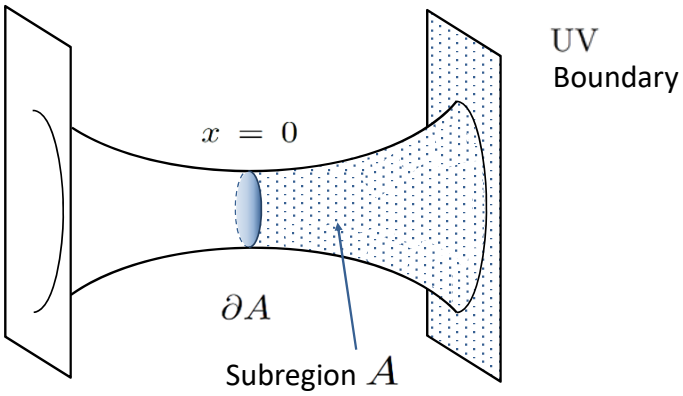
Testing QNEC in boundary wormhole

RHS: S_{out} Half-space Entanglement Entropy



- wish to evaluate Entanglement Entropy $S(A)$ for subregion A

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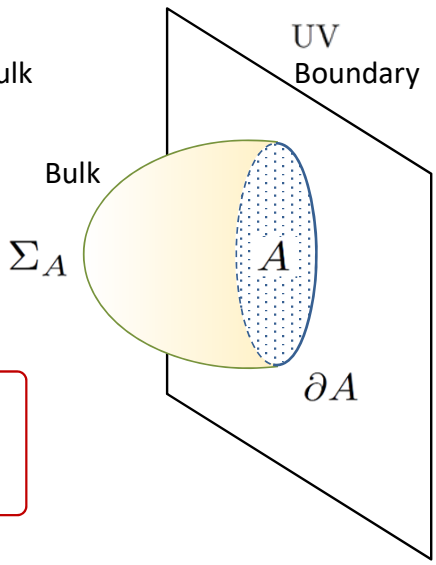
Testing QNEC in boundary wormhole

RHS: S_{out} as a Holographic Entanglement Entropy

Σ_A : Minimal area surface in Bulk anchored to ∂A

$\mathcal{A}(\Sigma_A)$: Area of Σ_A



Ryu-Takanayagi formula

$$S(A) = \frac{\mathcal{A}(\Sigma_A)}{4G}$$

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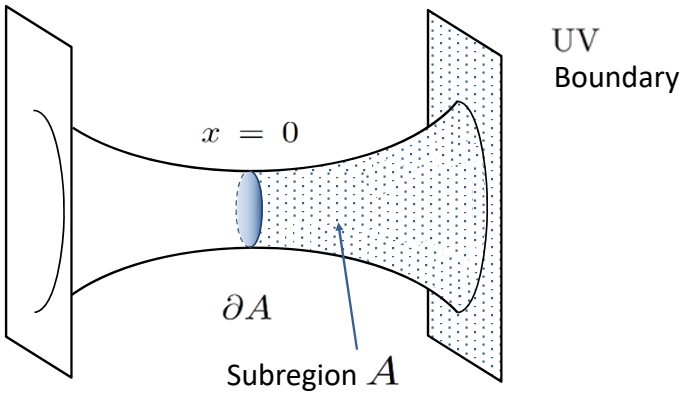
Testing QNEC in boundary wormhole

RHS: S_{out} Half-space Entanglement Entropy



- wish to evaluate Entanglement Entropy $S(A)$ for subregion A

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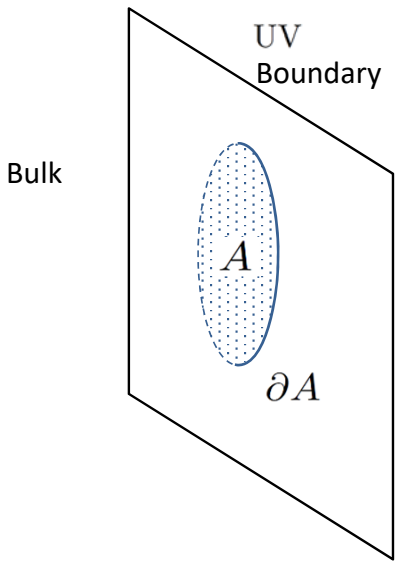
Testing QNEC in boundary wormhole

RHS: S_{out} as a Holographic Entanglement Entropy



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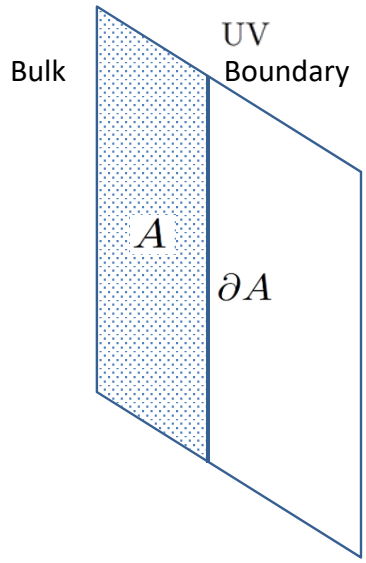
Testing QNEC in boundary wormhole

RHS: S_{out} as a Half-space Entanglement Entropy



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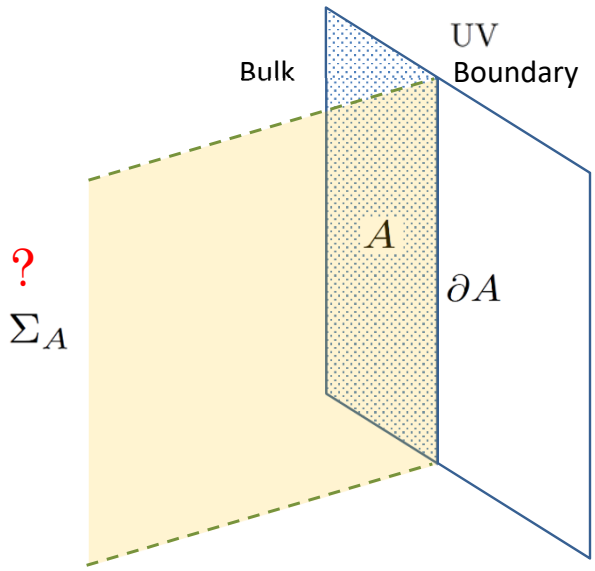
Testing QNEC in boundary wormhole

RHS: S_{out} as a Half-space Entanglement Entropy



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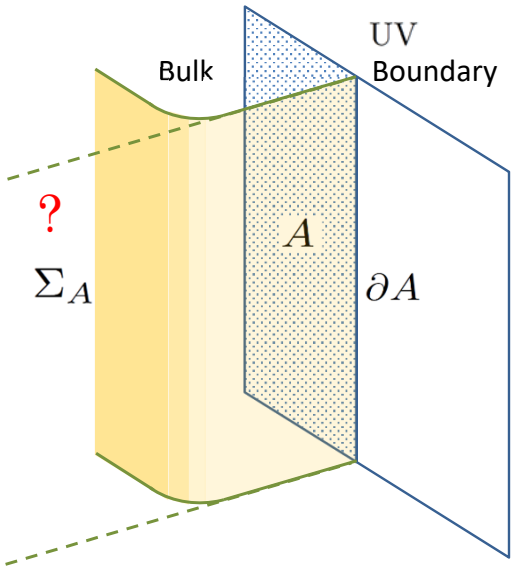
Testing QNEC in boundary wormhole

RHS: S_{out} as a Half-space Entanglement Entropy



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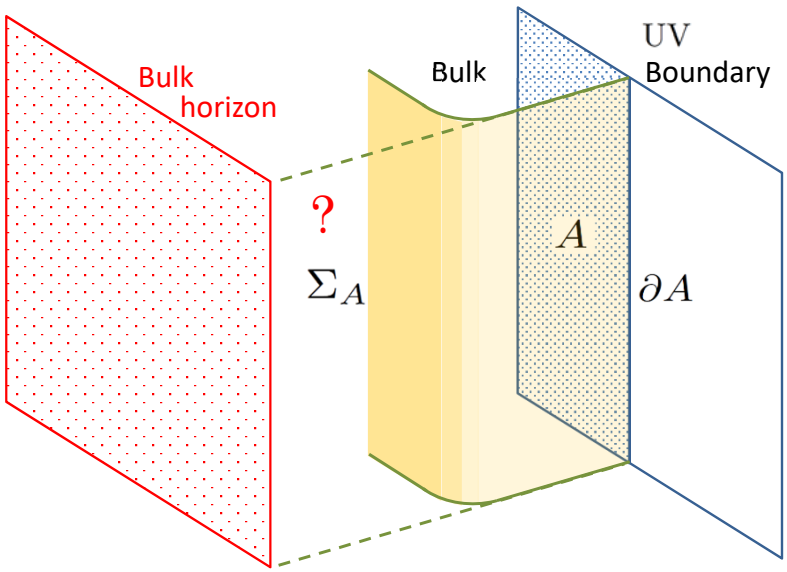
Testing QNEC in boundary wormhole

RHS: S_{out} as a Half-space Entanglement Entropy



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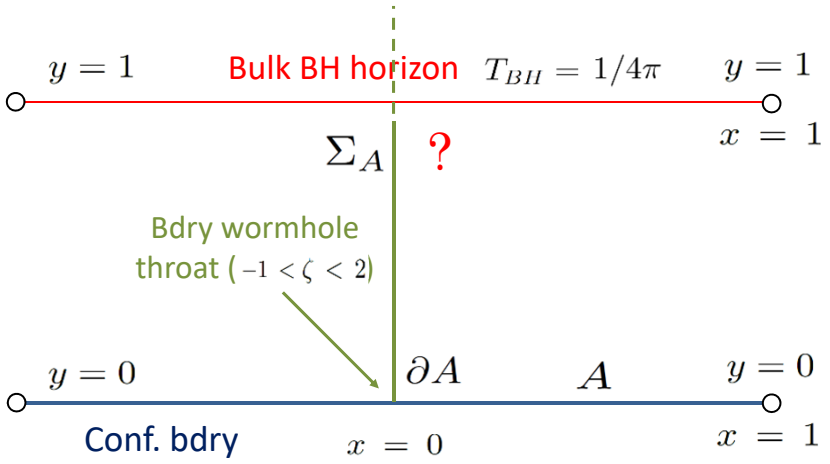
Testing QNEC in boundary wormhole

RHS: S_{out} as a Holographic Entanglement Entropy



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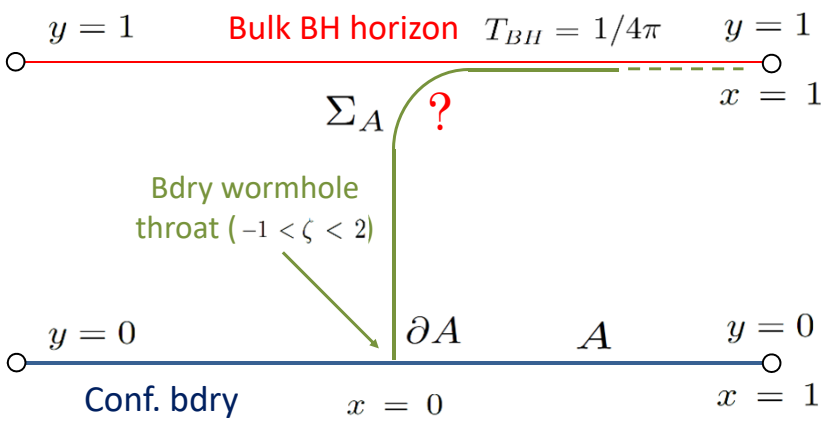
Testing QNEC in boundary wormhole

RHS: S_{out} as a Holographic Entanglement Entropy



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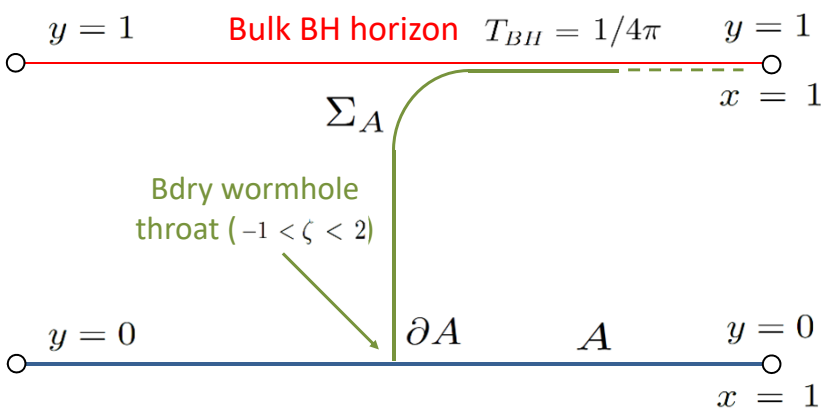
Testing QNEC in boundary wormhole

RHS: S_{out} as a Holographic Entanglement Entropy



$\mathcal{A}(\Sigma_A)$: Area of Σ_A

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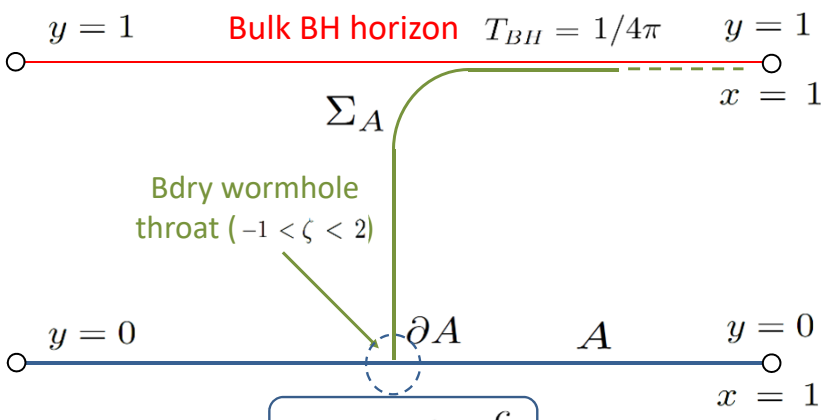
Testing QNEC in boundary wormhole

RHS: S_{out} as a Holographic Entanglement Entropy



UV regularization $\boxed{\mathcal{A}_{UV} := \mathcal{A} - \frac{c}{\epsilon}}$ - subtracting counter-term prop. to Area of ∂A

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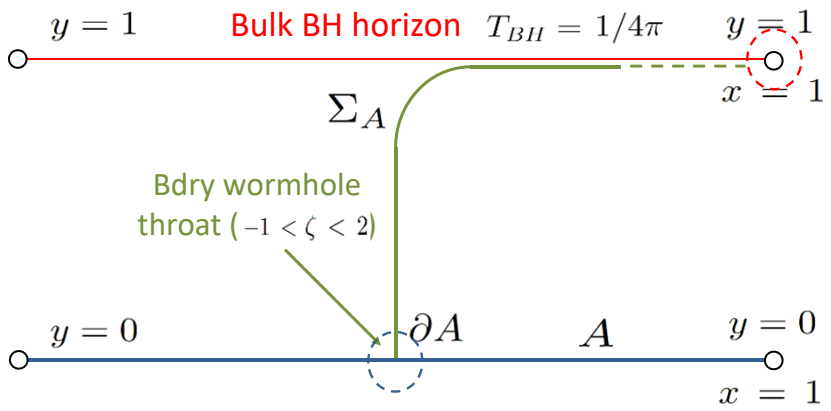
Testing QNEC in boundary wormhole

RHS: S_{out} as a Holographic Entanglement Entropy



IR regularization

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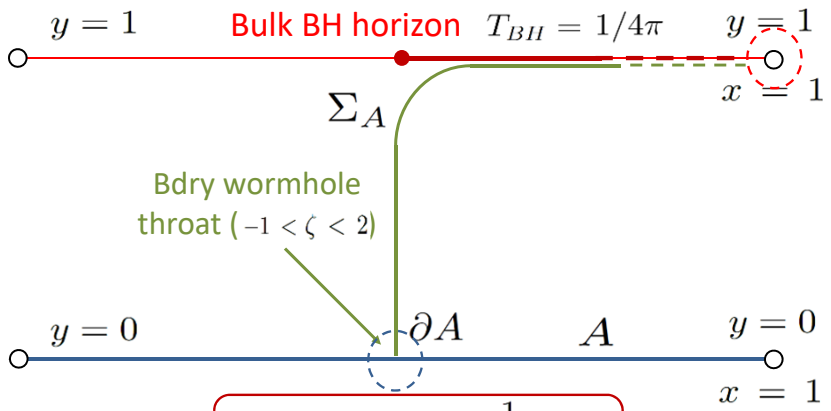
Testing QNEC in boundary wormhole

RHS: S_{out} as a Holographic Entanglement Entropy



IR regularization $\boxed{\mathcal{A}_{reg} := \mathcal{A}_{UV} - \frac{1}{2}\mathcal{A}_{BH}}$ - subtracting half of black-hole area

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Testing QNEC in boundary wormhole

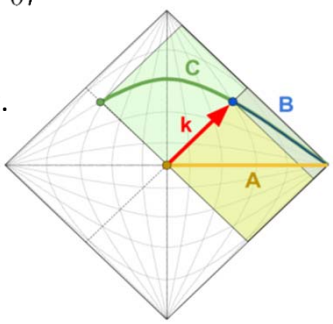
RHS: S_{out} as a Holographic Entanglement Entropy

$$S_{out} = \frac{\mathcal{A}_{reg}}{4\pi G}$$

In our boundary wormhole, QNEC $\langle T_{kk} \rangle \geqslant \frac{\hbar}{2\pi} S''_{out}$ reduces to

$$Q := 2\pi \langle T_{kk} \rangle - \frac{1}{32\pi G\sqrt{1+\zeta}} \frac{\delta^2 \mathcal{A}_{reg}}{\delta r^2} \geqslant 0$$

Null variation along k^a is taken at throat.



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Testing QNEC in boundary wormhole

RHS: S_{out} as a Holographic Entanglement Entropy

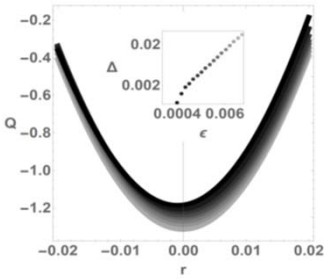
$$S_{out} = \frac{\mathcal{A}_{reg}}{4\pi G}$$

In our boundary wormhole, QNEC $\langle T_{kk} \rangle \geqslant \frac{\hbar}{2\pi} S''_{out}$ reduces to

$$Q := 2\pi \langle T_{kk} \rangle - \frac{1}{32\pi G\sqrt{1+\zeta}} \frac{\delta^2 \mathcal{A}_{reg}}{\delta r^2} \geqslant 0$$

Numerical evaluation of the QNEC near wormhole throat shows

Violation of QNEC $Q < 0$



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Summary and discussions

We constructed a 4D AdS-black hole w/ a 3D wormhole on the boundary.

Using holographic methods, we tested QNEC in our boundary wormhole spacetime and found violation of QNEC.

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Summary and discussions

QNEC has been proven for some cases.

All the proofs so far made however do not consider thermal states, where IR degrees play a role.

The result may depend on the way of IR-regularization. c.f. Leichenauer 1808.05961 under some symmetry assumptions

Our IR-regularization captures half-space EE, isolating the purely entanglement part from the thermal part.

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