Ice Thickness in the Arctic Sea

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Abstract

The fact that the Arctic Sea is covered with ice has tremendous importance on the cooling of ocean and, therefore, on the environmental condition of the earth. The solar pond effect is one of the key mechanism of determining the ice thickness. Considerations based on dynamical system theory leads to an empirical method of determining factors affecting ice thickness. The method consists of the construction of multidimensional space representing the environmental condition from the observational data. Then, the principal component analysis can be used to derive differential equations of the global change, if time derivatives in various time scales are included in constructing the multidimensional space.

Key words: The Arctic Sea, Ice Thickness, Global Ocean Circulation.

1 Introduction

Global warming is the most serious environmental problem, even if acid rain, ozone hole and other problems cannot be disregarded as well. Polar regions have been a global cooling system since the time of formation of oceans. For this reason, the arctic ice thickness is one of the best indicators of the global environmental condition. Physics determining the ice thickness, however, is not completely known. In that case, the role of the indicator and the conditions indicated may be reversed so that the factors affecting ice thickness can be looked for by an appropriate choice of data set in the principal component analysis under the guidance of the dynamical system theory. Before entering into the problem, however, we discuss here physical conditions of the Arctic Sea and the role on the global environment very briefly.

The importance of the Arctic Sea on the global environment is primarily manifested by the global ocean circulation which keeps the main body of deep ocean in low temperatures. Salty cold water produced in the Arctic Sea pours out to the bottom of the Atlantic, circulates through the Indian or the Pacific Ocean, and returns to the Arctic Sea in time scale of 10^3 years without changing the temperature appreciably. Low temperature in the Arctic Sea is due to the high latitude but the most important effect is the high reflectivity (90%) of snow surface on the permanent ice. The temperature is below the freezing point in the surface layer but is a little warmer in deeper layers. This is convectively unstable situation in uniformly salty water unlike fresh water showing the highest density at about 4°C. In the case of the Arctic Sea, fresh water is supplied by rivers of the surrounding continents and by rain, and spreads over saltier water below. The salinity gradient can prevent convection until mixing takes place with underlying subcritical convective instability. The latter instability is a nonlinear phenomenon of longer time scale depending very much on local conditions. The stabilization of convection by salinity gradient is known as one of the important technics in the solar pond technology. Without this effect, temperature in the surface layer will be warmer and the permanent ice will not be permanent. Solar energy absorbed in the Sea will be ten times more, the temperature will be increased, and the bottom of ocean will probably be dead (no oxygen) without global ocean circulation. The Arctic Sea is using the solar pond effect to produce salty cold water which starts traveling at the north end of the Atlantic, and circulates throughout main global oceans.

Now, in order to quantify the above picture, we have to have some methodology, and the principal component analysis seems to be the most appropriate. The underlying assumption of the principal component analysis is that the global environment constitutes a big dynamical system and that sufficiently large number of different data should describe all the degrees of freedom involved in the system. Therefore, the rate of change of the ice thickness can be represented by other measured quantities. In other words, one can in principle find the differential equation governing the ice thickness by observing environmental scientific experiments performed by the global environment itself. The second principal component analysis which includes the rate of change of observational data as an additional coordinate of enlarged multidimensional space representing the system can be introduced for isolating factors affecting ice thickness in the form of differential equations.

2 Primary Source Data

The earth atmosphere is quite abnormal in the sense that carbon dioxides are abnormally less abundant in contrast to abnormally rich content of oxygen molecules compared with the neighboring planets, Venus and Mars. The key factors of the abnormality are among others solar radiation, ocean, and life. Global environment problems should, therefore, be studied not only for short human time scale but also on much longer evolutionary time scales. The elementary coupling mechanisms leading to the evolutionary global change, however, should be sought also within the arctic polar environment data, and this should be possible if the arctic environment behaves as a chaotic dynamical system.

To know the factors affecting ice thickness is to determine the conditions of ocean which couple strongly with global environmental conditions. Since the solar radiation is one of the key factors, the solar radiation flux monitored at position p at time t, R(p,t), should be taken as one of the primary source data of the problem, where p = 1, 2, 3,..., p^* and t = 1, 2, 3,..., t^*. Then, salinity S(p,t), temperature T(p,t), arctic ice thickness U(p,t), and ocean current velocity V(p,t) near the surface may be taken as the primary data concerning ocean. From the world of life in polar regions, sea weed W(p,t) and marine bioluminescence X(p,t) may be taken. Interactions with other regions should be represented by temperature Y(p,t), pressure Y(p,t), pressure Z(p,t), humidity A(p,t), wind speed B(p,t), and ozone column density C(p,t). Ideally, all the primary data are to be monitored every day in every 100km square, say. But the completeness of data is not necessary in practice, as will be discussed later.

3 Embedding in Multidimensional Space

The factors affecting ice thickness is naturally sought within the polar region. A question, however, arises whether the global environment can be described only with the polar region data, since the polar region is not a closed system. The answer is probably "yes". The global environment is a chaotic dynamical system with strong interactions among different regions. The dynamical behavior outside the polar region is mapped on the polar region data, as far as the ice thickness is concerned. For the analysis of a chaotic dynamical system, however, we need to describe the data in more than (2n+1) dimensions, n being the fractal dimension of the system.

We are now interested in a time scale longer than a year. So, we take the yearly running average of primary source data Q(p,t) and denote it again by Q(p,t). Also, we have the yearly running average of daily amplitude of variation $\Delta_d Q$(p,t) and of yearly amplitude of variation $\Delta_y Q$ (p,t). Therefore, we have a set of data [Q(p,t),$\Delta_d Q$(p,t), $\Delta_y Q$(p,t)] out of the primary source data Q(p,t). Addition of $\Delta_d Q$ and $\Delta_y Q$ are for visualizing the nonlinear coupling with short time variations. These amplitudes of variations are expected to play the role of diffusivities in the amplitude equation of Q to be derived in the 2nd principal component analysis. We also introduce rate of change $\partial_t Q$ and acceleration $\partial_t^2 Q$ or equivalently rates of change in 2 time scales $\partial_{\tau 1}$ and $\partial_{\tau 2}$ or equivalently Q(p,t+τ) and Q(p,t+2τ). Thus, we have a set of data:

$$[Q, \partial_t Q, \partial_t^2 Q, \Delta_d Q, \partial_t \Delta_d Q, \partial_t^2 \Delta_d Q, \Delta_y Q, \partial_t \Delta_y Q, \partial_t^2 \Delta_y Q]$$

for all $Q = R, S, T, ..., C$. We have 108 elements constituting 108 dimensional phase space in which p*t* cases in time and positions are plotted. The distribution of points plotted in that phase space is a complete description of the system.

4 Utility of PCA

The distribution of p*t* points in the 108-dimensional phase space can be analyzed to give the fractal dimension n from the count $C(r)$ ($\propto r^n$), $C(r)$ being number of pairs with the mutual distance shorter than r. But, the principal component analysis gives an alternative way of determining the dimension of the system. The distribution of the data points are approximated by an ellipsoidal distribution in multidimensional space, and the direction of the longest major axis gives the 1st principal component, the next longest the 2nd and so forth. For the data normalized to have unit r.m.s. variance, principal components with their eigen-values of the correlation matrix much smaller than unity are relatively unimportant and can be neglected. Principal components having significant ($>$ or ~ 1) eigen-values are considered as governing the system, and number of significant principal components is considered as the effective dimension of the system.

Principal components are linear combinations of the source variables with the coefficients (direction cosines of the ellipsoid axes) corresponding to the correlations between the principal components and the source variables. Interpretation of the major principal components should give the model deciding which combinations of physical variables are governing the ice thickness.

Two remarks may be added here. By means of improved principal component analysis (Unno and Yuasa, 1991), even fragmentary data can be utilized constructively without decreasing the accuracy of the results. The second remark is that if the source data form a too big correlation matrix to handle, we can work on several subsets of source data and then the resulting set of significant principal components will be the source data of the 2nd PCA.

5 Second PCA

The idea of the 2nd PCA can be modified to provide the way of a mechanism hunting procedure. Let us consider that the 1st PCA has been able to sort out all the n significant principal components without using time derivatives of data as source data. This means that the number of measured quantities are more than the degree of freedom of the system. Then, the addition of the rate of change of the ice thickness in the 2nd PCA should be able to isolate the correlation between the rate of change of the ice thickness and the principal components hitherto obtained, since there should be no new significant principal components. The result will be a differential equation describing the rate of change of ice thickness with other measured quantities. Physical interpretation of the equation may follow. The method can be applied to other studies of natural history as well as to quantitative studies in social and medical sciences (cf. Benigni and Giuliani, 1993).

References

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