MIMO state-space model identification using step responses

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Abstract

In this paper, we propose a deterministic off-line identification method [1] that obtains a state-space model by using input and output data with steady state values. The method is composed of the method [2] zeroing the $0 \sim N$-tuple integral values of output error of single-input single-output transfer function model and Ho-Kalman's method [3]. The method has an assumption that plant system matrix A satisfies $|A| < 1$ because the method utilizes the Taylor series expansion of the plant. We prove that the method can be applied to plants without the assumption $|A| < 1$. The feature is that the method can identify by using step responses, the derived state-space model is emphasized in low frequency range and, consequently, this is suitable for a mechanical system identification in which noise and vibration are undesirable. Numerical simulations of multi-input multi-output system identification are illustrated.

Key words: identification, state-space model, step response, multi-input multi-output, mechanical system

1 Introduction

The deterministic off-line identification method [1] that obtains a state-space model by using input and output data with steady state values has an assumption that plant system matrix A satisfies $|A| < 1$ because the method utilizes the Taylor series expansion of the plant. In this paper, we prove that the method can be applied to plants without the assumption $|A| < 1$.

Properties that an useful identification method should have are as follows:

1. Small noise or vibration in identification experiment, that is, a step signal is better than a pseudo white signal.

2. The derived model is suitable for servo design, that is, the model is emphasized in low frequency range [4].

3. A reduced order model can be directly obtained from input and output data.

4. State-space model can be obtained.

5. Robustness against unmodelled dynamics.

6. Robustness against disturbance.

As for 4, in multi-input multi-output (MIMO) identification, single-input single-output (SISO) type of identification method requires the orders of the numerators and denominators of all the elements in a transfer function matrix. Consequently, this often causes over-parameterization. On the other hand, methods that derive state space model do not cause this problem.

As for the above mentioned properties $1 \sim 6$, evaluations of some representative identification methods are shown in Table 1 that includes our subjective views. A least squares, a step response and a frequency response methods are representative as a deterministic off-line identification method. Since an input signal, such as M series signal, used in a least squares method is required to satisfy P. E. condition, it often causes resonance in a mechanical plant which consequently results in large noise or vibration. And so is sine wave used in a frequency response method. Although a step signal causes noise or vibration at the moment when it is supplied, they are attenuated asymptotically. Therefore a step signal is practical for identification, because it causes noise or vibration only at the beginning during a long time interval of measuring, it can be generated easily by switching on and off, and it omits the adjustment of

Table 1: Evaluations of representative identification methods

	1	2	3	4	5	6
Least squares method[5], [6]	×	×	×	×	×	△
Frequency response method	×	×	×	×	×	△
Sub-space method[7], [8]	◯	×	◯	◯	△	△
Kosaka's method[2]	◯	◯	◯	×	◯	×
Proposed method	◯	△	◯	◯	△	×

sampling period required in using an M series signal. A conventional least squares method employing a step response restricts the order of an identified model [5]. In the case of servo system, it is desirable that the modelling error is small in low frequency range [4], but least squares methods have a tendency to reduce the error in high frequency range [6]. Sub-space based methods [7, 8] need to obtain a sufficiently large size of, for example, Hankel matrix before deriving its reduced order model, and also use a least squares method in obtaining some system parameters. The method [2] zeroing the $0 \sim N$-tuple integral values of output error of single-input single-output model has the properties 1, 2, 3, 5 but derives transfer function model.

In this paper, we propose a deterministic off-line identification method [1] that obtains a state-space model by using input and output data with steady state values. The method is composed of the method [2] zeroing the $0 \sim N$-tuple integral values of output error of single-input single-output transfer function model and Ho-Kalman's method [3]. The method has an assumption that plant system matrix A satisfies $|A| < 1$ because the method utilizes the Taylor series expansion of the plant. We prove that the method can be applied to plants without the assumption $|A| < 1$. The feature is that the derived state-space model is emphasized in low frequency range.

The paper is organized as follows. In section 2, assumptions are made for a plant, and the identification method is stated. In section 3, the properties of the identified model are stated. In section 4, numerical simulations are illustrated to indicate the effectiveness of the proposed method.

2   Identification algorithm

In this section, assumptions are made for a plant, and the identification algorithm [1] is stated.

2.1   Plant to be identified

The plant to be identified is represented by the following linear continuous-time n_u-input n_y-output state-space system:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) + Du(t) \end{cases} \quad (1)$$

where t is time and $x(t) \in R^{n_A}$, $y(t) \in R^{n_y}$ and $u(t) \in R^{n_u}$ are the state, input and output vector, respectively.

2.2   Assumptions

We make the following assumptions for the system:

A1: $\det(A) \neq 0$

A2: C and A satisfy

$$\mathrm{rank} \begin{bmatrix} -CA^{-1} \\ -CA^{-2} \\ \vdots \\ -CA^{-(\alpha_y - 1)} \end{bmatrix} = n_A \quad (2)$$

where $\det(A)$ denotes the determinant of A and α_y is the integer that satisfies

$$\frac{n_A}{n_y} + 1 \leq \alpha_y < \frac{n_A}{n_y} + 2. \quad (3)$$

2.3   Identification algorithm

The number of acquired input and output data with steady state is n_u where those data are denoted as $u_i(t)$ and $y_i(t)(i = 1, 2, \cdots, n_u)$, respectively, and $u_i(t)$'s satisfy

$$\mathrm{d_{et}}\left([u_1(\infty) \ u_2(\infty) \cdots u_{n_u}(\infty)]\right) \neq 0. \quad (4)$$

From input and output data, $U(t)$ and $p_0(t)$ are defined as

$$U(t) = [u_1(t) \ u_2(t) \cdots u_{n_u}(t)]$$
$$[u_1(\infty) \ u_2(\infty) \cdots u_{n_u}(\infty)]^{-1}, \quad (5)$$
$$p_0(t) = [y_1(t) \ y_2(t) \cdots y_{n_u}(t)]$$
$$[u_1(\infty) \ u_2(\infty) \cdots u_{n_u}(\infty)]^{-1}. \quad (6)$$

The integer α_u and n_ρ are defined so as to satisfy

$$\frac{n}{n_u} \leq \alpha_u < \frac{n}{n_u} + 1, \tag{7}$$

$$n_\rho = \alpha_y + \alpha_u - 1 \tag{8}$$

where n means the order of the system matrix $\hat{A}$ in the model. ρ_0 is derived as

$$\rho_0 = p_0(\infty). \tag{9}$$

ρ_i's $(i = 1, 2, \cdots, n_\rho)$ are derived as the following equations:

$$p_1(t) = \int_0^t (p_0(\tau) - \rho_0 U(\tau)) d\tau \tag{10}$$

$$\rho_1 = p_1(\infty) \tag{11}$$

$$p_2(t) = \int_0^t (p_1(\tau) - \rho_1 U(\tau)) d\tau \tag{12}$$

$$\rho_2 = p_2(\infty) \tag{13}$$

$$\vdots$$

$$p_i(t) = \int_0^t (p_{i-1}(\tau) - \rho_{i-1} U(\tau)) d\tau \tag{14}$$

$$\rho_i = p_i(\infty). \tag{15}$$

$(\alpha_y n_y \times \alpha_u n_u)$ matrix H is defined as

$$H = \begin{bmatrix} \rho_1 & \rho_2 & \cdots & \rho_{\alpha u} \\ \rho_2 & \rho_3 & \ddots & \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{\alpha y} & & \cdots & \rho_{n\rho} \end{bmatrix}. \tag{16}$$

By using the singular value decomposition, H is decomposed as

$$H = U\Sigma V^T \tag{17}$$

where Σ is the diagonal matrix of which elements are the singular values of H. $(\alpha_y n_y \times n)$ matrix O and $(n \times \alpha_u n_u)$ matrix C_o are defined as

$$O = U(1 : \alpha_y n_y, 1 : n) \Sigma^{\frac{1}{2}} (1 : n, 1 : n) \tag{18}$$

$$C_o = \Sigma^{\frac{1}{2}} (1 : n, 1 : n) V (1 : \alpha_u n_u, 1 : n)^T \tag{19}$$

where MATLAB notation[8] is used. According to this notation, $A(a_1 : a_2, b_1 : b_2)$ means the matrix of which elements are those in the $a_1 \sim a_2$ columns and the $b_1 \sim b_2$ rows in A.

The estimated system parameter $(\hat{A}, \hat{B}, \hat{C}, \hat{D})$ is obtained by the following calculations:

$$O_u = O(1 : (\alpha_y n_y - n_y), 1 : n) \tag{20}$$

$$O_d = O((n_y + 1) : \alpha_y n_y, 1 : n) \tag{21}$$

$$\hat{A} = O_d^\dagger O_u \tag{22}$$

$$\hat{B} = \hat{A} C_o (1 : n, 1 : n_u) \tag{23}$$

$$\hat{C} = -O(1 : n_y, 1 : n) \hat{A} \tag{24}$$

$$\hat{D} = \rho_0 + \hat{C} \hat{A}^{-1} \hat{B} \tag{25}$$

where $\dagger$ denotes pseudo inverse matrix.

2.4 Another identification algorithm

Another algorithm to identify $\hat{A}$ by using C_o instead of O is stated here.

A2' is assumed instead of A2.

A2':A and B satisfies

$$\text{rank} \left[A^{-1}B, \ A^{-2}B, \cdots, \ A^{-(\alpha_u - 1)}B \right] = n_A \tag{26}$$

where α_u is the integer that satisfies

$$\frac{n_A}{n_u} + 1 \leq \alpha_u < \frac{n_A}{n_u} + 2. \tag{27}$$

Instead of (3), (20), (21) and (22), we respectively use

$$\frac{n}{n_y} \leq \alpha_y < \frac{n}{n_y} + 1 \tag{28}$$

$$C_u = C_o (1 : n, 1 : (\alpha_u n_u - n_u)) \tag{29}$$

$$C_d = C_o (1 : n, (n_u + 1) : \alpha_u n_u) \tag{30}$$

$$\hat{A} = C_u C_d^\dagger. \tag{31}$$

By using this algorithm, the same results are derived through the same procedure as the next section.

3 Properties of the model

In this section, it is shown that the model corresponds with the plant if $n = n_A$ without assumption $|A| < 1$.

As $U(t)$ has steady state from (4) and (5), by using the final value theorem, it follows that

$$d_{et} \left(\lim_{t \to \infty} U(t) \right) = d_{et} \left(\lim_{s \to 0} s\bar{U}(s) \right) \neq 0 \tag{32}$$

where s is a complex variable and $\bar{x}(s)$ means the Laplace transformation of $x(t)$. Therefore, using a matrix $\bar{U}_a(s)$ $(\det(\bar{U}_a(0)) \neq 0)$ of which elements are rational functions of s, $\bar{U}(s)$ can be expressed as

$$\bar{U}(s) = \frac{\bar{U}_a(s)}{s}. \tag{33}$$

From (5), (32) and (33), we obtain

$$
\begin{aligned}
\bar{U}_a(0) &= \lim_{s \to 0} \bar{U}_a(s) \\
&= \lim_{s \to 0} s\bar{U}(s) \\
&= \lim_{t \to \infty} U(t) \\
&= \lim_{t \to \infty} [u_1(t) \; u_2(t) \cdots u_{n_u}(t)] \\
&\quad [u_1(\infty) \; u_2(\infty) \cdots u_{n_u}(\infty)]^{-1} \\
&= [u_1(\infty) \; u_2(\infty) \cdots u_{n_u}(\infty)] \\
&\quad [u_1(\infty) \; u_2(\infty) \cdots u_{n_u}(\infty)]^{-1} \\
&= I^{n_u \times n_u}.
\end{aligned} \tag{34}
$$

By using the final-value theorem, we obtain

$$
\begin{aligned}
\rho_0 &= p_0(\infty) \quad \because)(9) \\
&= \lim_{t \to \infty} [y_1(t) \; y_2(t) \cdots y_{n_u}(t)] \\
&\quad [u_1(\infty) \; u_2(\infty) \cdots u_{n_u}(\infty)]^{-1} \quad \because)(6) \\
&= \lim_{s \to 0} s \left(D + C(sI - A)^{-1} B \right) \\
&\quad [\bar{u}_1(s) \; \bar{u}_2(s) \cdots \bar{u}_{n_u}(s)] \\
&\quad [u_1(\infty) \; u_2(\infty) \cdots u_{n_u}(\infty)]^{-1} \quad \because)(1) \\
&= \lim_{s \to 0} s \left(D + C(sI - A)^{-1} B \right) \frac{\bar{U}_a(s)}{s} \\
&\qquad\qquad\qquad \because)(5),\ (33) \\
&= \left(D - CA^{-1}B \right) \bar{U}_a(0) \\
&= D - CA^{-1}B. \quad \because)(34)
\end{aligned} \tag{35}
$$

ρ_1 is obtained in the same way as

$$
\begin{aligned}
\rho_1 &= p_1(\infty) \quad \because)(11) \\
&= \lim_{t \to \infty} \int_0^t (p_0(\tau) - \rho_0 U(\tau))\, d\tau \quad \because)(10) \\
&= \lim_{s \to 0} s\frac{1}{s} \left((D + C(sI - A)^{-1}B) \right. \\
&\quad \left. \bar{U}(s) - \rho_0 \bar{U}(s) \right) \quad \because)(1),\ (5),\ (6) \\
&= \lim_{s \to 0} \left((D + C(sI - A)^{-1}B) - (D - CA^{-1}B) \right) \\
&\qquad\qquad \frac{\bar{U}_a(s)}{s} \quad \because)(33),\ (35) \\
&= \lim_{s \to 0} \left(C\left((sI - A)^{-1} + A^{-1}\right)B \right) \frac{\bar{U}_a(s)}{s} \\
&= \lim_{s \to 0} \left(C(sI - A)^{-1}(sA^{-1})B \right) \frac{\bar{U}_a(s)}{s} \\
&= -CA^{-2}B. \quad \because)(34)
\end{aligned} \tag{36}
$$

Assume that $p_i(t)$ $(i \geq 1)$ satisfies

$$
\begin{aligned}
p_i(t) &= \int_0^t (p_0(\tau) - (\rho_0 + \rho_1\,\delta + \cdots \\
&\quad + \rho_{i-1}\delta^{i-1})U(\tau))(d\tau)^i
\end{aligned} \tag{37}
$$

$$
\rho_i = -CA^{-(i+1)}B \tag{38}
$$

where δ denotes a differential operator. $p_{i+1}(t)$ is yielded as

$$
\begin{aligned}
&p_{i+1}(t) \\
&= \int_0^t (p_i(\tau) - \rho_i U(\tau))\, d\tau \quad \because)(14) \\
&= \int_0^t \left(\int_0^t (p_0(\tau_1) - (\rho_0 + \rho_1\,\delta + \cdots + \right. \\
&\quad \left. \rho_{i-1}\delta^{i-1})U(\tau_1))(d\tau_1)^i - \rho_i U(\tau_2) \right) d\tau_2 \ \because)(37) \\
&= \int_0^t \left((p_0(\tau) - (\rho_0 + \rho_1\delta + \cdots + \rho_{i-1}\delta^{i-1})U(\tau)) \right. \\
&\quad \left. -\rho_i \delta^i U(\tau) \right)(d\tau)^{i+1} \\
&= \int_0^t \left(p_0(\tau) - (\rho_0 + \rho_1\delta + \cdots + \rho_{i-1}\delta^{i-1} + \rho_i\delta^i) \right. \\
&\quad \left. U(\tau) \right)(d\tau)^{i+1}
\end{aligned} \tag{39}
$$

and then ρ_{i+1} is yielded as

$$
\begin{aligned}
&\rho_{i+1} \\
&= \lim_{t \to \infty} p_{i+1}(t) \quad \because)(15) \\
&= \lim_{s \to 0} s\frac{1}{s^{i+1}} \left(\bar{p}_0(s) - (\rho_0 + \rho_1 s + \cdots + \rho_i s^i)\bar{U}(s) \right) \\
&\qquad\qquad \because)(39) \\
&= \lim_{s \to 0} \frac{1}{s^i} \left((D + C(sI - A)^{-1}B) \right. \\
&\quad \left. - (\rho_0 + \rho_1 s + \cdots + \rho_i s^i) \right) \frac{\bar{U}_a(s)}{s} \\
&\qquad\qquad \because)(1),\ (5),\ (6),\ (33) \\
&= \lim_{s \to 0} \frac{1}{s^{i+1}} \left((D + C(sI - A)^{-1}B) - \right. \\
&\quad \left. ((D - CA^{-1}B) - CA^{-2}Bs - \cdots - CA^{-(i+1)}Bs^i) \right) \\
&\quad \bar{U}_a(s) \qquad\qquad \because)(35),\ (36),\ (38) \\
&= \lim_{s \to 0} \frac{1}{s^{i+1}} \left(C(sI - A)^{-1}B + \right. \\
&\quad \left. CA^{-1}B + CA^{-2}Bs + \cdots + CA^{-(i+1)}Bs^i \right)\bar{U}_a(s) \\
&= \lim_{s \to 0} \frac{1}{s^{i+1}} \left(C((sI - A)^{-1} + \right. \\
&\quad \left. A^{-1} + A^{-2}s + \cdots + A^{-(i+1)}s^i)B \right)\bar{U}_a(s) \\
&= \lim_{s \to 0} \frac{1}{s^{i+1}} \left(C(sI - A)^{-1}(I + (sI - A) \right. \\
&\quad \left. (A^{-1} + A^{-2}s + \cdots + A^{-(i+1)}s^i))B \right)\bar{U}_a(s) \\
&= \lim_{s \to 0} \frac{1}{s^{i+1}} \left(C(sI - A)^{-1}(sA^{-1} + \right. \\
&\quad \left. (sI - A)(A^{-2}s + \cdots + A^{-(i+1)}s^i))B \right)\bar{U}_a(s) \\
&= \lim_{s \to 0} \frac{1}{s^{i+1}} \left(C(sI - A)^{-1}(s^2 A^{-2} + \right. \\
&\quad \left. (sI - A)(A^{-3}s + \cdots + A^{-(i+1)}s^i))B \right)\bar{U}_a(s) \\
&\vdots
\end{aligned}
$$

$$
\begin{aligned}
&= \lim_{s\to 0} \frac{1}{s^{i+1}} \left(C \left(sI - A\right)^{-1} \left(s^i A^{-i} + \right.\right. \\
&\qquad \left.\left. \left(sI - A\right)\left(A^{-(i+1)} s^i\right)\right) B\right) \bar{U}_a\left(s\right) \\
&= \lim_{s\to 0} \frac{1}{s^{i+1}} \left(C\left(sI - A\right)^{-1} \left(s^{i+1} A^{-(i+1)}\right) B\right) \bar{U}_a\left(s\right) \\
&= \lim_{s\to 0} \left(C\left(sI - A\right)^{-1} A^{-(i+1)} B\right) \bar{U}_a\left(s\right) \\
&= -CA^{-(i+2)} B. \quad \because)(34) \qquad (40)
\end{aligned}
$$

Therefore, (37) and (38) follow inductively. By substituting (38) for (16), we get

$$
H = \\
-\begin{bmatrix}
CA^{-2}B & CA^{-3}B & \cdots & CA^{-\alpha_u - 1}B \\
CA^{-3}B & CA^{-4}B & \ddots & \\
\vdots & \vdots & \ddots & \vdots \\
CA^{-\alpha_y - 1}B & & \cdots & CA^{-n_\rho - 1}B
\end{bmatrix}. \quad (41)
$$

If $n = n_A$, from (3), (7), (8), (17), (18), (19), A1, A2 and [3], it follows that

$$
H = OC_o \qquad (42)
$$

$$
O = \begin{bmatrix}
-CA^{-1} \\
-CA^{-2} \\
\vdots \\
-CA^{-\alpha_y}
\end{bmatrix} \qquad (43)
$$

$$
C_o = \left[A^{-1}B,\ A^{-2}B, \cdots,\ A^{-\alpha_u}B\right]. \qquad (44)
$$

From (20), (21) and (43), we get

$$
O_d = O_u A^{-1}. \qquad (45)
$$

From (45), the pseudo inverse matrix $O_d{}^\dagger$ is defined as

$$
O_d{}^\dagger = \left(A^{-1}\right)^T \left(A^{-1}\left(A^{-1}\right)^T\right)^{-1} \left(O_u^T O_u\right)^{-1} O_u^T. (46)
$$

From A1, A2, (22) and (46), we derive

$$
\begin{aligned}
\hat{A} &= O_d{}^\dagger O_u \\
&= \left(A^{-1}\right)^T \left(\left(A^{-1}\right)^T\right)^{-1} \left(A^{-1}\right)^{-1} \\
\therefore \hat{A} &= A. \qquad (47)
\end{aligned}
$$

From A1, (7), (23), (44) and (47), we obtain

$$
\begin{aligned}
\hat{B} &= \hat{A} C_o\left(1:n, 1:n_u\right) \\
&= \hat{A} A^{-1} B \\
\therefore \hat{B} &= B. \qquad (48)
\end{aligned}
$$

From (3), (24), (43) and (47), we derive

$$
\begin{aligned}
\hat{C} &= -O\left(1:n_y, 1:n\right)\hat{A} \\
&= -\left(-CA^{-1}\right)\hat{A} \\
\therefore \hat{C} &= C. \qquad (49)
\end{aligned}
$$

From (25), (35), (47), (48) and (49), we obtain

$$
\begin{aligned}
\hat{D} &= \rho_0 + \hat{C}\hat{A}^{-1}\hat{B} \\
&= D - CA^{-1}B + \hat{C}\hat{A}^{-1}\hat{B} \\
&= D - CA^{-1}B + CA^{-1}B \\
\therefore \hat{D} &= D. \qquad (50)
\end{aligned}
$$

From the above, the model corresponds with the plant.

Next we consider that the method emphasizes low frequencies. By using the Taylor series expansion around $s = 0$, the plant (1) is expressed as

$$
\begin{aligned}
\bar{y}\left(s\right) &= \left(D + C\left(sI - A\right)^{-1}B\right)\bar{u}\left(s\right) \\
&= \left(\left(D + C\left(-A\right)^{-1}B\right) - C\left(-A\right)^{-2}Bs + \cdots \right. \\
&\qquad \left. + C\frac{\left(-(i-1)\right)!}{(i-1)!}\left(-A\right)^{-i}Bs^{i-1} + \cdots\right)\bar{u}\left(s\right) \\
&= \left(\left(D - CA^{-1}B\right) - CA^{-2}Bs - \cdots \right. \\
&\qquad \left. - CA^{-i}Bs^{i-1} - \cdots\right)\bar{u}\left(s\right). \qquad (51)
\end{aligned}
$$

The coefficients in (51) are derived from low to high order $(0 \sim n_\rho)$ in turn, from which A, B, C and D are obtained. That is, the coefficeients higher than n_ρ are ignored. This leads to emphasize low frequencies because low order coefficients represent the characteristics in low frequency range.

4   Numerical simulations

In this section, the proposed method is applied to identify 2-input and 2-output system by using step responses of the system. The model is compared with conventional sub-space method [8].

4.1   System and simulation setting

The plant to be identified is as follows:

$$
G\left(s\right) = \\
\begin{bmatrix}
10\frac{200}{250}\frac{(s+10)\left(s^2 + 5^2\right)}{(s+2)\left(s^2 + 2\cdot 0.1\cdot 10s + 10^2\right)} & -\frac{1}{s+2} \\
1 & 10\frac{25}{50}\frac{(-s+50)}{\left(s^2 + 2\cdot 0.2\cdot 5s + 5^2\right)}
\end{bmatrix} (52)
$$

The order of this system $n_A = 5$ and $\max|A| > 1$. The simulator is set up as follows:

Sampling time: 0.01

Data length : 60[s]

Input signal for identification : the following step signal

$$u_1(t) = (s(t) \quad 0)^T \tag{53}$$
$$u_2(t) = (0 \quad s(t))^T$$

where $s(t)$ is a step function.

In the case of the sub-space method, both step signal (54) and M series signal (55) are used for identification

$$u(t) = \begin{cases} (s(t) \quad 0)^T & , \ 0 < t \le 60[s] \\ (0 \quad s(t))^T & , \ 60 \le t \le 120[s] \end{cases} \tag{54}$$

$$u(t) = (m_1(t) \quad m_2(t))^T \ , \ 0 \le t \le 120[s] \tag{55}$$

where $m(t)$ is a M series function with amplitude 1 and the minimum period 11 samples.

4.2 Results

The plant is identified for each $n(= 1, 2, \cdots, 5)$. The model identified by the proposed method becomes stable with $n = 1, 3, 5$. The step responses of the (1×1) elements of these models are shown in Fig. 1. From Fig. 1, the proposed method emphasizes low frequencies.

In the case of the sub-space method, the model becomes stable only with $n = 5$ when step input

is used for identification. The step response of this (1×1) element of the model is shown in Fig. 2. From Fig. 2, the model is not good even if $n = n_A$. When M series input is used, the models become stable with $n = 4, 5$. The step responses of these (1×1) element of the models are shown in Fig. 3. From Fig. 3, the sub-space method does not emphasize low frequencies.

5 Conclusion

The assumption as plant system matrix A satisfies $|A| < 1$ of the deterministic off-line state-space model identification method [1] has been taken off. The method can obtain the model from input and output data with constant steady state. It has been verified by numerical simulations that the proposed method emphasizes low frequencies.

This method is suitable to identify mechanical systems without genarating noise or vibration because the method can identify MIMO system by using step responce.
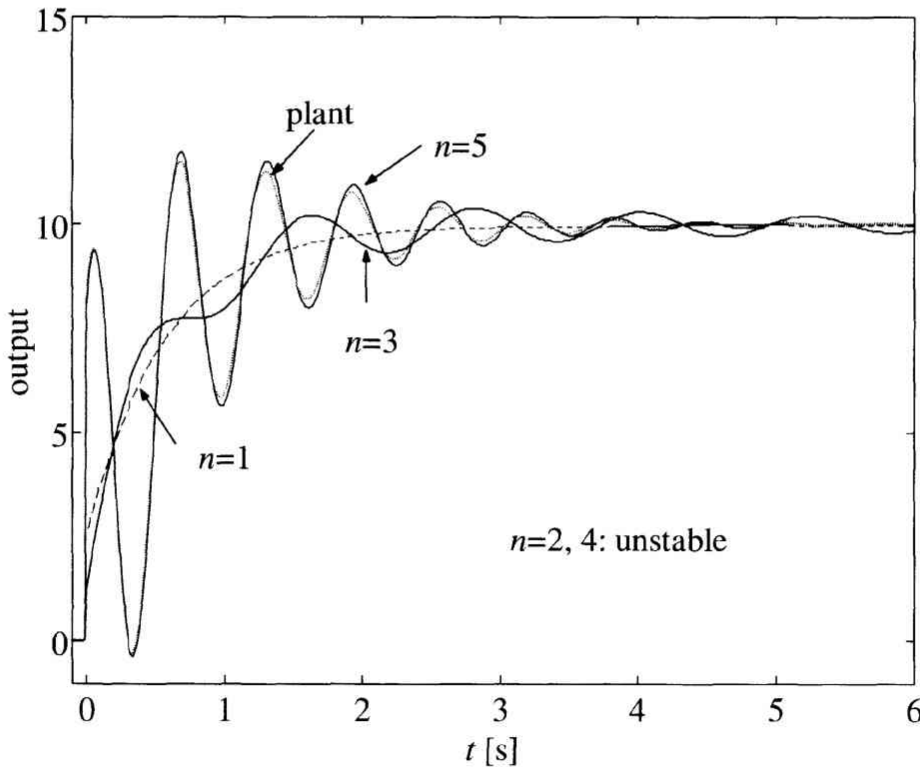

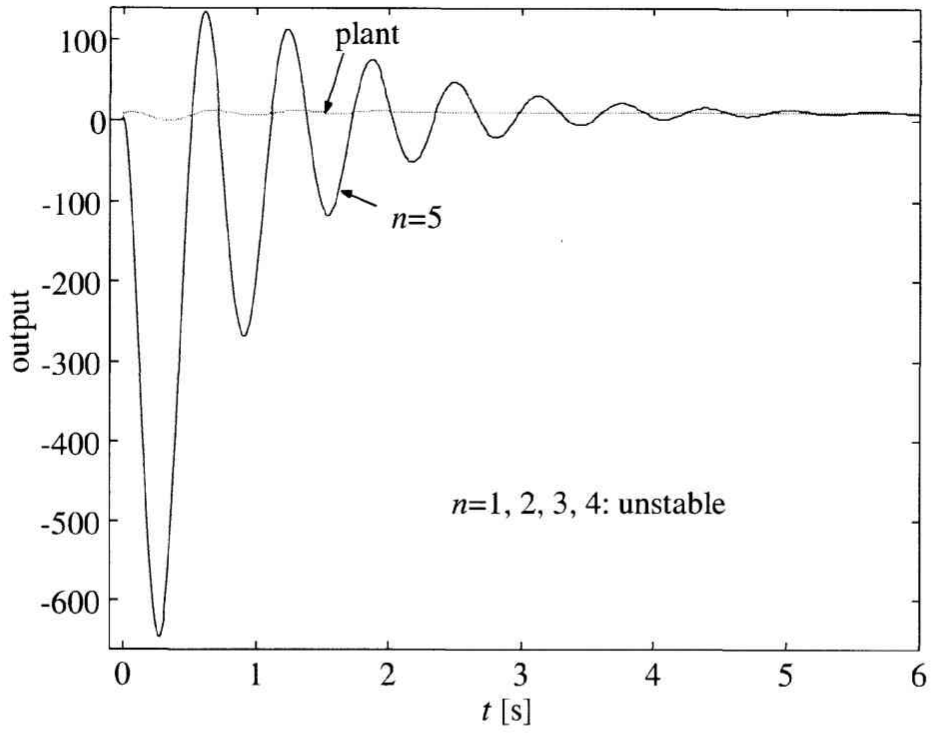
Figure 1: Step responses (proposed meth.)

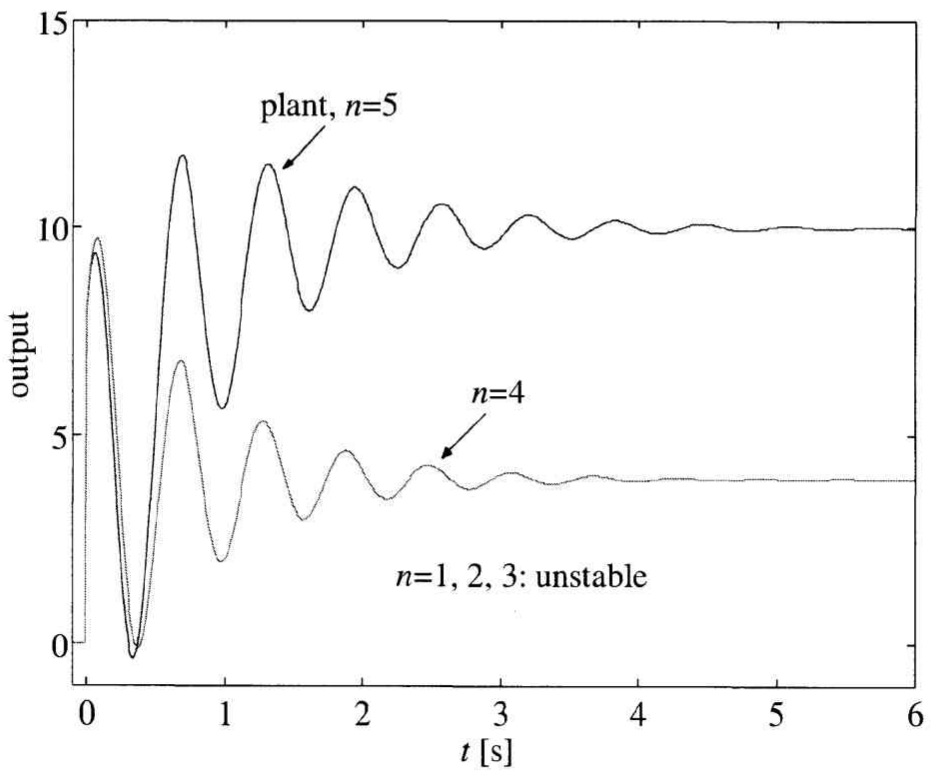Figure 2: Step responses (sub-space meth. with step input)

Figure 3: Step responses (sub-space meth. with M series input)

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