Gravitational Collapse of Cold Dark Matter in Early Universe using Schrödinger Equation

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Abstract

We have numerically solved a non-linear Schrödinger equation which describes the growth of the density perturbation in an expanding universe. We have explored the characteristics of this Schrödiger equation method because this method has numerous advantage over N-body simulation method for the study of gravitational collapse of Cold dark matter.

Key Words: Cosmology: theory – large-scale structure of the Universe – non-linear Schrödinger equation.

1 Introduction

Development of studies on early universe has made it clear that our universe consists of 73% of dark energy, 23% of dark matter and 4% of baryon matter.

The dark energy is a jargon to indicate a form of energy described by the cosmological constant Λ in the equation of general relativity. It is an important problem to study the character of dark energy, but we treat it phenomenologically in this paper.

The dark matter is a jargon to indicate a form of matters which interacts with ordinary matters effectively only through gravity. The dark matter is dark because it does not have electro-magnetic interaction. We do not yet succeed to discriminate between various candidates for the dark matter. It may be a relic of fermions such as neutralino which are non-relativistic at the epoch of frozen interaction. It may be a relic of condensate bosons such as axion. It may be black holes.

In order to study the nature of cold dark matter (CDM), we must first know how the CDM distributes in astrophysical objects, i.e., we must know how the cosmological density perturbation grows in the history of the universe.

As Widrow and Kaiser (1993) pointed out, when we observe an object of the scale of L through the window of the scale of η, time evolution of the state of an object in phase space is described by a non-linear Schrödinger equation, if $\lambda_{\rm deb} << \eta << L$, where $\lambda_{\rm deb} = \frac{\hbar}{p}$ is the deBlogie wave length. If CDM is blackhole, it is adequate to use N-body simulation, but if CDM is subatomic particle, it is more adequate to use the Schrödinger equation.

As Jones (1999) pointed out, in order to study the origin of scaling and lognormal intermittency in the galaxy distribution, it is important to study various wave mechanical methods including the non-linear Schrödinger equation.

Although we can not carry the prospect to make an accurate simulation, because we must deploy impossibly fine grids of the order of deBlogie length, the situation is the same for N-body calculation in which we must deploy impossible many particles.

In this paper, we pursue the possibility of the calculation of the structure formation in our universe through the Schrödinger equation.

2 Basic Equations

The equation of motion for CDM in the universe with cosmic constant Λ is given by the equation of continuation

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho u_i)}{\partial x_i} = 0, \tag{1}$$

the Euler equation

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial \phi}{\partial x_i}, \tag{2}$$

and the Poisson equation

$$\frac{\partial^2 \phi}{\partial x_i \partial x_i} = 4\pi G\rho - \Lambda c^2. \tag{3}$$

where ρ is the density, u_i is the velocity of cold dark matter and ϕ is the gravitational potential. When the motion of the universe is uniform and isotropic, we can assume that

$$u_{0i} = H(t)x_i, \quad \rho_0 = \rho_0(t), \quad \phi_0 = \phi_0(r,t), \tag{4}$$

and then we have the basic equation for the expansion of the universe

$$\frac{1}{\rho_0}\frac{\partial \rho_0}{\partial \tau} = -3 \tag{5}$$

$$\frac{1}{H}\frac{\partial H}{\partial \tau} = q - 1 \tag{6}$$

where the cosmic time is given by the red shift parameter z by

$$\frac{d\tau}{dt} = H, \quad e^{-\tau} = 1 + z \tag{7}$$

and the deceleration parameter q is given by density parameters for CDM $\Omega_{\rm m}$ and cosmological constant Ω_Λ as

$$q = -\frac{1}{2}\Omega_{\rm m} + \Omega_\Lambda, \tag{8}$$

$$\Omega_{\rm m} = \frac{8\pi G\rho_0}{3H^2}, \quad \Omega_\Lambda = \frac{\Lambda c^2}{3H^2}. \tag{9}$$

The Hubble parameter H and density parameters are functions of cosmic time, and the values at present time is

$$H_0 = 71 \pm 4\,\mathrm{km\,s^{-1}\,Mpc^{-1}},$$
$$\Omega_{\rm m} = 0.27,$$
$$\Omega_\Lambda = 0.73.$$

From Eq.(5) and Eq.(6), we have

$$\frac{1}{\Omega_\Lambda}\frac{d\Omega_\Lambda}{d\tau} = 2q + 2, \tag{10}$$

$$\frac{1}{\Omega_{\rm m}}\frac{d\Omega_{\rm m}}{d\tau} = 2q - 1. \tag{11}$$

We have derived this expansion equation in the scheme of Newtonian mechanics, but this equation is the same which is derived in terms of the general relativity (Tomita and Hayashi, 1963). As we have

$$\frac{d(\Omega_\Lambda + \Omega_{\rm m})}{d\tau} = 2q(\Omega_\Lambda + \Omega_{\rm m} - 1), \tag{12}$$

if the universe is flat

$$\Omega_\Lambda + \Omega_{\rm m} = 1 \tag{13}$$

at initial time, it continues to be flat after that time.

For the displacement of the density σ, the velocity u'_i and the gravitational potential ϕ' from the uniform isotropic background, which is defined by

$$\rho = \rho_0(1 + \sigma(x,t)), \tag{14}$$

$$u_i = u_{0i} + u'_i(x,t), \tag{15}$$

$$\phi = \phi_0 + \phi'(x,t), \tag{16}$$

we obtain from Eq.(1), Eq.(2) and Eq.(3)

$$\left(\frac{\partial}{\partial \tau} + x_j \frac{\partial}{\partial x_j}\right)\sigma = -\frac{\partial}{\partial x_i}\left[(1+\sigma)\frac{u'_i}{H}\right], \tag{17}$$

$$\left(\frac{\partial}{\partial \tau} + x_j \frac{\partial}{\partial x_j} - q\right)\frac{u'_i}{H} + \frac{u'_j}{H}\frac{\partial}{\partial x_j}\frac{u'_i}{H} = -\frac{1}{H^2}\frac{\partial \phi'}{\partial x_i}, \tag{18}$$

$$\frac{\partial^2 \phi'}{\partial x_i \partial x_i} = 4\pi G\rho_0\sigma. \tag{19}$$

Transforming to co-moving coordinate

$$x'_i = x_i e^{-\tau} \tag{20}$$

$$\tau' = \tau \tag{21}$$

and carrying out variable transformations

$$\frac{u'_i}{H} = e^{\tau}u''_i, \quad \phi' = e^{2\tau}4\pi G\rho_0\phi'', \tag{22}$$

we obtain, after abbreviating primes

$$\frac{\partial \sigma}{\partial \tau} + \frac{\partial}{\partial x_i}[(1+\sigma)u_i] = 0, \tag{23}$$

$$(\frac{\partial}{\partial \tau} + 1 - q)u_i + u_j \frac{\partial}{\partial x_j} u_i = -3(q+\Omega_\Lambda)\frac{\partial \phi}{\partial x_i}, \tag{24}$$

$$\frac{\partial^2 \phi}{\partial x_i^2} = \sigma. \tag{25}$$

We assume that the velocity field is irrotational, so we can introduce velocity potential θ by

$$u_i = \frac{\partial \theta}{\partial x_i}. \tag{26}$$

We carry out Madelung transformation (Madelung, 1926) using parameter ν with the dimension of length/time which describes the velocity dispersion of CDM,

$$\Psi = a e^{\frac{i\theta}{\nu}}, \tag{27}$$

$$a^2 = 1 + \sigma \tag{28}$$

then, we get the non-linear Schrödinger equation (Spiegel, 1980)

$$i\frac{\partial \Psi}{\partial \tau} + \frac{\nu}{2}\Delta\Psi = V\Psi, \tag{29}$$

$$V = \frac{\nu}{2}\frac{\Delta a}{a} + (1-q)\frac{\theta}{\nu} + \frac{3}{\nu}(q+\Omega_\Lambda)\phi. \tag{30}$$

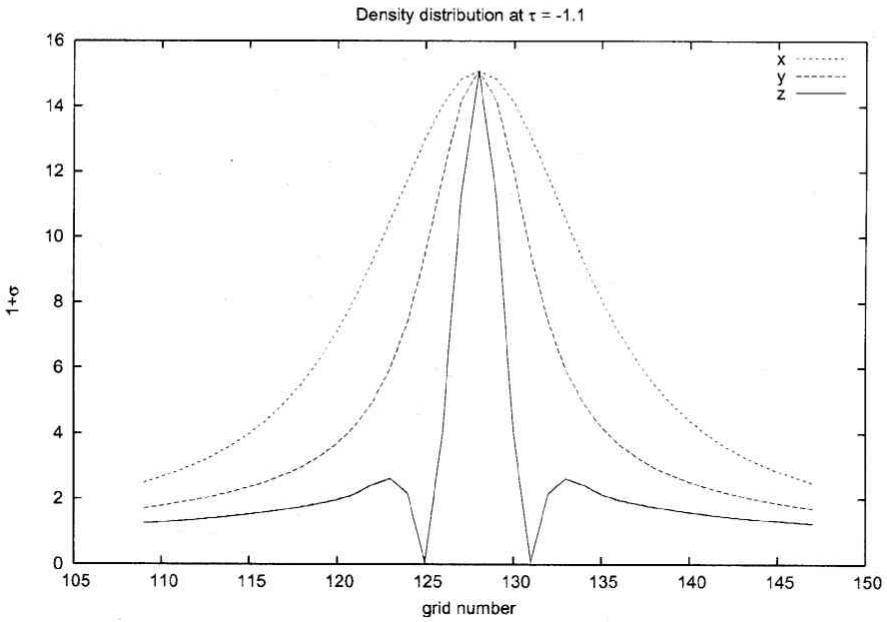

Figure 1: Density distribution at $z = 2$. Abscissa shows the grid number, the center of which is 128. Ordinate shows the density, background value of which is 1.

3 Numerical Simulation

The solution of Schrödinger equation

$$(i\frac{\partial}{\partial \tau} - K)\Psi = V\Psi, \quad K = -\frac{\nu}{2}\Delta \tag{31}$$

is given by

$$\Psi(\tau) = -i\int_0^\tau e^{-K(\tau-\tau')}V(\tau')\Psi(\tau')d\tau' + e^{-K\tau}\Psi(0). \tag{32}$$

When $|\tau V| << 1$, we have a trapezium formula

$$[1 + iK\tau][1 + \frac{i}{2}\tau V(\tau)]\Psi(\tau) = [1 - \frac{i}{2}\tau V(0)]\Psi(0). \tag{33}$$

This is an implicit integration method with an accuracy of second order, and we can solve the

Schrödinger equation stably. We put initial condition at $\tau = \log_e(1+z)$, $z = 1000$ of the form

$$\Phi = -\epsilon \frac{1}{(1+a_1)(1+a_2)} \times \tag{34}$$

$$\cos(x)(\cos(y) + a_1 \cos(2y))(\cos(z) + a_2 \cos(2x)), \tag{35}$$

$$a = 1 + \frac{1}{6}\Delta\Phi, \tag{36}$$

$$\theta = -\frac{\nu}{3}\Phi. \tag{37}$$

We give a typical asymmetry (Bardeen et. al., 1986)

$$a_1 = 0.111, \quad a_2 = 0.304. \tag{38}$$

Coles and Spencer made a simulation of one dimensional collapse in Schrd̈inger Scheme.They compared their result with Zel'dovich approximation. Their result shows that sufficiently before the shell crossing, the result agrees each other. Near the shell crossing, high density peak of Zel'dovich approximation does not seen, and after the crossing time, intense wave with short wave length arises.

In Figure 1, we show a result of numerical simulation for $\nu = 10^{-3}$. The grid size of this simulation is 256^3 and we show only the central part. In the density distribution, hollows appear in the z direction. This is an effect caused by the quantum pressure $\Delta a/a$. The width of the central peak decreases when the parameter ν decreases.

In this calculation, we include only CDM. We are now making a calculation, in which CDM is coupled with the baryon fluid.

4 Summary

In this paper, we have checked the availability of the Schrödinger equation method in the study of early universe, and we showed that this method is suitable to study the dynamics of CDM. We showed that before CDM collapse completely, void appears around the high density peak which cancelates completely the background density of the universe. This result is the characteristic feature of the Schrd̈inger method, which may have something to do with the lognormal intermittency of the galaxy distribution.

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