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# Stock Price Paths in Markets with Short-Sales Constraints Populated by Behavioral Traders*

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**Abstract** Investor confidence affects financial markets. Information, noise, market frictions cause investor confidence to influence security prices, leading to a price different from the rational expectations value. This paper presents a simple theoretical model of asset prices where investor confidence is allowed to differ across traders, and across time — depending on observed outcomes. The presence of short-sales constraints causes asset prices to behave asymmetrically: short-run returns display reversal after good news, but momentum after bad news. This changes somewhat if investor confidence varies because of biased self-attribution: good news causes returns to exhibit short-run momentum and long-run reversal.

**Key words** behavioral finance, overconfidence, underconfidence, overreaction, momentum
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# 1 Introduction

One of the most widely discussed behavioral phenomena concerned with financial asset markets is that of overconfidence, i.e. a tendency for people to put too much weight in their own judgments, to believe their own point of view to be more accurate than it actually is when considered objectively. While in this paper it is mainly individual investors who are thought of as the acting economic agents, it has been found that experts, armed with sophisticated models, actually exhibit more overconfidence than laymen ! Economist Robert J. Shiller (2000a) 〔4〕 provides a comprehensive account of investor opinion surveys carried out over a period spanning some two decades, indicating a pervasive tendency for both individual and institutional investors to be overconfident. Overconfidence is usually considered as a static phenomenon; for example, an investor is assumed to perceive the variance of an information signal he obtains to be lower than it truly is. In this paper, a plausible dynamic interpretation of overconfidence is also considered: an investor confident in his prior valuation of a risky asset (based, say, on a research done by herself on a company) is liable to update his valuation differently depending on the type of new information. Overconfidence makes him overreact to news confirming his estimate and underreact to news that contradicts his valuation. Such a behavioral pattern is called in the cognitive psychology literature *biased self-attribution*. It causes individuals to attribute successes to their own qualities and failures to chance. Overconfidence and biased self-attribution are static and dynamic counterparts; self-attribution causes individuals to learn to be overconfident rather than converging to an accurate self-assessment. Shiller (2000b) 〔5〕 presents evidence that investor confidence does vary through time, based on investor opinion polls in the US and Japan.

In this article, interaction between overconfident and underconfident traders in a financial market setting is analyzed. While overconfidence is a well-documented pervasive pattern of human behavior, underconfidence seems to be a less salient phenomenon. However, it may become more understandable why underconfidence, and generally heterogeneous confidence among different traders, is employed in our model, if we observe its connection with noise. In his influential essay, Black (1986) [1] remarks that "people sometimes trade on noise as if it were information". He further asserts that "people who trade on noise are willing to trade even though from an objective point of view they would be better off not trading". In today's world of information flooding, distinguishing noise from valuable information is an extremely difficult task. This observation motivates the approach in this paper based on investor psychology. The investors who are in possession of a piece of information can never be sure that they are actually trading on information rather than noise. It is possible the information has already been reflected in prices. Trading on that sort of information would be just like trading on noise. It is therefore plausible to say that investors who are uncertain as to the quality and relevance of their information are underconfident.

The effect of short-sales constraints is also examined in a behavioral finance context. Selling short can be expensive. In order to sell short, one must borrow the stock from a current owner, and this stock lender may charge a fee to the short seller. In addition to these direct costs, there are other costs and risks associated with shorting, such as the risk that the short position will have to be involuntarily closed at a loss due to a recall of the stock loan. In addition, legal and institutional constraints inhibit or prevent investors from selling short. Finally, some market participants seem to behave as though they were facing considerable shorting costs, even though they are not. In financial economics, these impediments and costs are collectively referred to as "short sales constraints". We attempt to analyze the impact of short sales constraints on the equilibrium prices in presence of confidence-biased investors.

The formal analysis is contained in section 2; it commences with a simple analytical model, where the possibility of equilibrium prices being equal to their rational values is investigated in a market populated by less than rationally behaving investors. The overall flow of this paper is organized as follows: section 2 contains the basic model with some extensions; we discuss the relevance of our findings and some extensions in section 3.

## 2 Investor Confidence and Short-Sales Constraints

This section presents the formal analysis of asset prices with emphasis on investor confidence, short-sales constraints and the interplay of these two factors. The basic structure is presented in section 2.1. It covers the derivation of investors' demand functions, equilibrium prices, as well as a simple examination of the properties of prices, namely its relationship with a rational expectations equilibrium price. Section 2.2 introduces short-sales constraints and presents the consequences of their presence for the short-run behavior of asset returns. Outcome-dependent confidence, based on biased self-attribution, is then introduced in section 2.3, along with some implications for the performance of asset prices. This is an extension of one of the models from Daniel, Hirshleifer, and Subrahmanyam (1998) 〔2〕, with the addition of short-sales constraints.

### 2.1 Two Types of Traders with Opposing Biases: Security Price Behavior

We begin the formal analysis with a simple model of asset prices with two types of boundedly rational investors present in the market. The first type overestimates the accuracy of their information whilst the second type of investors underestimates the precision of theirs. Let us start the analysis with the characterization of the model structure. There is one risk-free asset with constant payoff equal to unity and one risky stock with net supply normalized to zero. There are three dates: $t = 0$, $t = 1$, $t = 2$. At $t = 2$, the stock pays a terminal dividend equal to $F$. It is assumed to be normally distributed according to $F \sim \mathcal{N}(0, \sigma_F^2)$. At $t = 1$, all traders receive a noisy signal about the risky asset's intrinsic value; the signal may thus be regarded as a public one:

$$s_1 = F + \epsilon, \tag{1}$$

where $\epsilon \sim \mathcal{N}(0, \sigma_\epsilon^2)$. The signal precision is given by the reciprocal of its variance, $1/\sigma_\epsilon^2$. Random variables $F$ and $\epsilon$ are independently distributed. At $t = 0$, the price is simply equal to its prior mean, $P_0 = 0$. Investors misperceive the precision of the information they receive; such a bias is modeled in the following way: an investor indexed with $k$ holds a belief about the distribution of her signal summarized by $s_k = F + B_k\epsilon$, where $k = c, d$. Overconfident investors, indicated with the subscript "$c$" believe that their signal is more accurate than it actually is, i.e. $0 < B_c < 1$ ; thus they believe its distribution to be "too tight". On the other hand, underconfident investors, denoted by "$d$" believe the distribution of their signal to be "too loose", i.e. $B_d > 1$. We also allow the case of fully rational expectations, so that $B_d \geq 1$. A possible justification for this structure is a situation where different investors get their information — which is the same from an objective point of view — from different sources: one reliable and the other questionable, or "untrustworthy". Then one group will believe the precision of their signal to be more accurate than it truly is, whilst the other group will think their signal contains too much noise. Such an arrangement results effectively in a structure that is possible to be characterized by overconfidence and underconfidence. Let the subset of investor population consisting of overconfident traders be denoted by $\lambda$, so that the number of underconfident traders is $1 - \lambda$, where $0 < \lambda < 1$.

All investors have CARA (Constant Absolute Risk Aversion) utility functions with equal risk tolerance coefficient $\gamma$:

$$\mathrm{E}[U(W_k)] = \mathrm{E}[-\exp(-\frac{W_k}{\gamma})] \quad \text{for} \quad k = c, d. \tag{2}$$

With normal distributions, this implies in effect a mean-variance utility function. In a multiperiod model, wealth depends on investor decisions in all the periods. Unfortunately, the general solution for such a problem is quite complex; however, for our purposes it will be sufficient to focus on "myopic" behavior: traders are assumed to focus only on the immediate period, and so decisions are independent across periods. This effectively ignores any inter-period linkages but does allow the problem to be analyzed tractably.

The wealth at the final date for each trader is the sum of the initial wealth $W_0$ and the gain derived from the two types of assets. Since the payoff of the risk-free asset is always equal to unity, it follows that for trader $k$ $(k = c, d)$ :

$W_k = W_0 + D_k(F - P)$, where $D_k$ is trader $k$'s demand for the risky asset and $P$ is its price. The trader $k$'s maximization problem is therefore:

$$\max_{D_k} \mathrm{E}_k[W_k|s] - \frac{\mathrm{Var}[W_k|s]}{2\gamma} \quad \text{s.t.} \quad W_k = W_0 + D_k(F - P) \quad \text{for} \quad k = c, d. \tag{3}$$

By standard properties of normal variables it follows that:

$$\mathrm{E}[F|s] = \mathrm{E}[F] + \frac{\mathrm{Cov}[F, s]}{\mathrm{Var}[s]}(F + \epsilon)$$

and

$$\mathrm{Var}[F|s] = \mathrm{Var}[F] - \frac{\mathrm{Cov}[F, s]\mathrm{Cov}[F, s]}{\mathrm{Var}[s]},$$

which in the present model becomes:

$$\mathrm{E}_k[F|s] = \frac{\sigma_F^2}{\sigma_F^2 + B_k^2\sigma_\epsilon^2}(F + \epsilon) \quad \text{for} \quad k = c, d; \tag{4}$$

$$\mathrm{Var}_k[F|s] = \frac{\sigma_F^2 B_k^2 \sigma_\epsilon^2}{\sigma_F^2 + B_k^2\sigma_\epsilon^2} \quad \text{for} \quad k = c, d, \tag{5}$$

where $B_k = B_c$ for overconfident traders and $B_k = B_d$ for underconfident traders. Furthermore, solving the above maximization problem yields the following demand functions:

$$D_k = \frac{\gamma[\alpha_k(F + \epsilon) - P]}{\beta_k}, \tag{6}$$

where $\alpha_k = \frac{\sigma_F^2}{\sigma_F^2 + B_k^2\sigma_\epsilon^2}$ and $\beta_k = \mathrm{Var}_k[F|s] = \frac{\sigma_F^2 B_k^2 \sigma_\epsilon^2}{\sigma_F^2 + B_k^2\sigma_\epsilon^2}$.

It can be seen that $\alpha_c > \alpha_d$ and $\beta_c < \beta_d$, resulting in $|D_c| > |D_d|$. To put it another way, overconfident traders' higher conditional mean and lower conditional variance result in them taking larger positions in the risky stock. The assumption of differing signal distributions leads to an equilibrium, which is not a rational expectations equilibrium. The model's equilibrium is thus characterized by the investors' optimal demands as given in (6) and by the market clearing condition equating total demand with total supply:

$$\lambda D_c + (1-\lambda) D_d = 0. \tag{7}$$

Substituting appropriate demand functions into the last equation results in the equilibrium price being equal to:

$$P_1 = \frac{\lambda \alpha_c \beta_d + (1-\lambda)\alpha_d \beta_c}{\lambda \beta_d + (1-\lambda)\beta_c}(F+\epsilon). \tag{8}$$

Having obtained the equilibrium price, we now turn our attention to the analysis of how this price relates to one that would be obtained under proper, neutral confidence; in doing so, the emphasis is applied at the issue of whether the above price can be equal to the "rational" price. In other words, the goal is to check if differing biases might cancel each other out leading to prices being close or equal to those in case of only rational investors being present.[1] To examine the possibility of the price being equal to its corresponding rational expectations value, let us first observe that this price would be equal to

$$P_1^r = \alpha_r (F+\epsilon) = \frac{\sigma_F^2}{\sigma_F^2 + \sigma_\epsilon^2}(F+\epsilon), \tag{9}$$

where the superscript "$r$" indicates the rational price. This would be the case if all the investors correctly estimated the precision of their signals, that is, if $B_c = B_d = 1$. Equating the last two formulas, $P = P_r$, yields after some algebra a condition necessary for the two biases to cancel each other out, leading to a rational equilibrium asset price:

$$P = P_r \iff 1 = \lambda \frac{1}{B_c} + (1-\lambda)\frac{1}{B_d}. \tag{10}$$

(1) One of leading arguments against behavioral finance is the assertion that different biases of quasi-rational agents cancel each other out in equilibrium and thus have no effect on prices and other variables of interest.

The above equation has some intuitive properties. Whether the price can attain its rational expectations value, will depend on the interplay of the three parameters: $\lambda$, $B_c$, and $B_d$. It can be seen that it is only in a special case that the price can attain its rational value. If the first term on the right-hand side of equation (10) prevails, overconfident investors' dominance will show in the price being above its absolute rational level; the opposite happens when the underconfident investors dominate the market. The asset price depends on the extent of investor confidence and the relative fractions of the two trader types in the whole population.

Proposition 1. *The relationship between the attained equilibrium price and the price in a model with only rational traders present is characterized by the following condition:*

$$1 \lesseqqgtr \lambda \frac{1}{B_c} + (1-\lambda)\frac{1}{B_d} \iff P \gtreqqless P_r. \tag{11}$$

Concentrating for the moment on the overconfident fraction of the trader population, it follows that the price cannot be rational if $\lambda > B_c$. Observe that the above condition can be rewritten in terms of the relative population fractions of both trader types as

$$\frac{1-\lambda}{\lambda} \lesseqqgtr \frac{\frac{1}{B_c}-1}{1-\frac{1}{B_d}} \iff P \gtreqqless P_r. \tag{12}$$

Thus, for any $B_d > 1$, we shall have:

Corollary 1. *The price of a risky asset in a market populated by overconfident and underconfident traders is larger than its rational counterpart if the fraction of overconfident traders is larger than the overconfident traders' bias, that is, if $B_c < \lambda$.*

It is worthwhile to examine some special cases. First, assume that the two biases are symmetric in the following sense: $B_d = 2 - B_c$, i.e. the overconfident overestimate the precision of their information to the same extent as the underconfident underestimate theirs. In this case, the necessary condition for the price to be rational is $B_c = 2\lambda$. But the price will not be rational if $\lambda > B_c$, and so the price can only attain its rational value if $\lambda < 1/2$.

Observation 1. *If the imperfectly rational investors' biases are symmetric, price cannot be equal to its rational value if there are more overconfident traders than underconfident ones.*

As a second special case, assume that the only biased group are the overconfident, that is $B_d = 1$. In this case, it follows clearly from the above discussion that the price will not be equal to its rational counterpart, as the necessary condition for this to happen, namely (12), becomes $B_c = 1$, while we have $0 < B_c < 1$.

Observation 2. *The price cannot attain its rational value if overconfident investors interact with rational ones — it will always overreact to new information, as long as overconfident traders are present in the market.*

Irrational investors will thus affect prices, similarly to the oft-cited model in De Long, Shleifer, Summers and Waldmann (1990) [3]. However, while in their model irrational investors have first-order erroneous beliefs — with respect to an asset's expected value, our traders only misperceive precision of their information.

### 2.2 Short-Sales Constraints

We now turn to the next part of the analysis, where the presence of short-sales constraints is assumed. Assume for the moment that the signal at $t = 1$ was positive. Whether fully rational or underconfident in the precision of their signals, the $1-\lambda$ fraction of such investors will be pushed out of the market by the overconfident investors if short-sale constraints are introduced into the model above. This is because the necessary condition for not overconfident traders to stay out of the market (this being equivalent to their valuation of the asset being lower than the prevailing price), that is $\alpha_d(F + \epsilon) < P_1$, is fulfilled for all allowed parameter values. The price in this case will be set by the overconfident traders, and it will always lie above the rational price level. On the other hand, let us assume the news at $t = 1$ was adverse and the signal was negative. Since the overconfident — by overestimating the precision of their signal — overreact to information, their valuation will now be below the market price and it is them now, it turn, who will be pushed out of the market by the other traders.

In the presence of short-sales constraints, the demand functions of the two types of investors have to be revised and will now be given by:

$$D_k = \max\left\{\frac{\gamma[\alpha_k(F+\epsilon)-P]}{\beta_k}, 0\right\}, \quad \text{for} \quad k=c,d. \tag{13}$$

The implications for prices follow immediately; they are summarized in the following statement:

Proposition 2. *At $t=1$, the risky asset price can fall in one of two distinct regions:*

1. *The price is set by the overconfident traders and the underconfident sit out of the market when the signal is positive. It is given by:*

$$P_1^c = \alpha_c(F+\epsilon) = \frac{\sigma_F^2}{\sigma_F^2 + B_c^2\sigma_\epsilon^2}(F+\epsilon). \tag{14}$$

2. *The price is set by the underconfident traders and the overconfident sit out of the market when the signal is negative. It is given by*

$$P_1^d = \alpha_d(F+\epsilon) = \frac{\sigma_F^2}{\sigma_F^2 + B_d^2\sigma_\epsilon^2}(F+\epsilon). \tag{15}$$

*The price will be equal to its unbiased value in a special case when $B_d=1$, i.e. when there was a negative signal and perfectly rational traders push the overconfident traders out of the market.*

3. *No matter if the signal is positive or negative, the fraction of overconfident or underconfident investors present in the market has no effect on equilibrium prices: the price $P$ does not depend on $\lambda$.*

It is clear that at $t=1$, $|P_c| > |P_d|$. Prices display asymmetry — they overreact to good signals and underreact to bad ones. This differs from Daniel, Hirshleifer, and Subrahmanyam (1998) 〔2〕, where the equilibrium price is symmetric in that it always overreacts to private signals, no matter if good or bad. Figure 1 presents

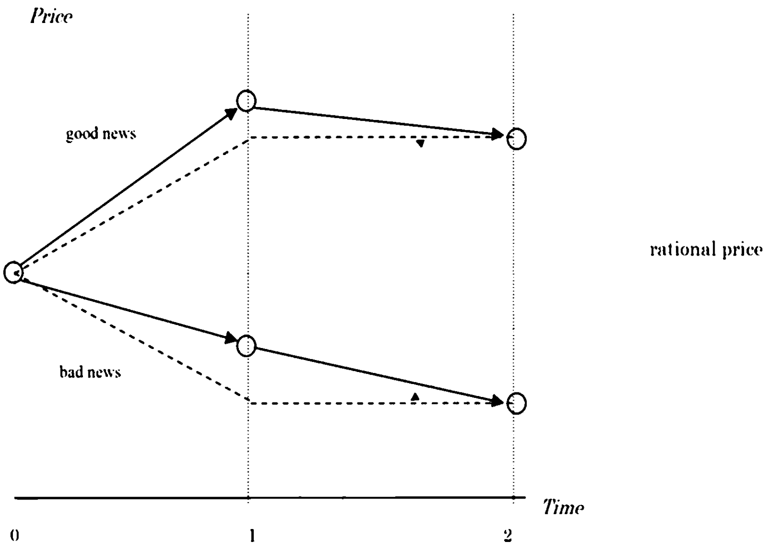


**Figure 1: Price behavior as a function of time with differing investor confidence and short-sales constraints. At time $t = 1$, equilibrium price is seen to display overreaction to good signals and underreaction to bad signals. As a consequence, short-run returns are positively autocorrelated in the bad news region, negatively autocorrelated in the good news region.**

this situation in a simple graphical example.

Let us recall that at $t = 0$, when investors had identical prior beliefs, the price was equal to zero, its prior mean, and notice further that at $t = 2$ the asset value becomes commonly known and thus will trivially be equal to $F$. This, combined with the results of the last proposition, allows us to draw conclusions regarding the short-term behavior of asset prices under short-sales constraints. We thus have the following corollary.

Corollary 2. *Prices exhibit momentum in the bad news region:* $\mathrm{Cov}[(\Delta P_2, \Delta P_1)|\epsilon < 0] > 0$, *and reversal in the good news region:* $\mathrm{Cov}[(\Delta P_2, \Delta P_1)|\epsilon > 0] < 0$, *where* $\Delta P_t = P_t - P_{t-1}$ *for* $t = 1, 2$.

### 2.3 Outcome-Dependent Confidence

Thus far, investor confidence only mattered at a single date. If there are more trading dates and information events in between the initial time $t = 0$ and the terminal announcement of the asset's true value, investor confidence might change as a result of actions and their outcomes. In this section, the dynamic counterpart of overconfidence, termed biased self-attribution is assumed to affect investor behavior. To analyze the effects of such behavior, a version of the model of Daniel, Hirshleifer, and Subrahmanyam (1998) 〔2〕 is considered, with the major divergence being the presence of short-sales constraints. A few adjustments to the previous model are thus introduced. Assume that the payout of the terminal dividend is postponed until a new date $t = 3$. There is a new public signal $s_2$ at $t = 2$; as in Daniel, Hirshleifer, and Subrahmanyam (1998) 〔2〕, it is assumed to be pure noise (This modeling device is for tractability purposes and should be understood as the limiting case of the signal being in fact correlated to the fundamental, when the correlation approaches zero). It can be equal to either $-1$ or $+1$; the probability of $s_2 = +1$ is exogenously given. All the traders are short-sales constrained. The underconfident investors do not react to new information. However, if the overconfident investors see a favorable signal at $t = 2$ after they previously received also a positive signal, they update their perceived precision of their $t = 1$ signal further, to $s_{1c} = F + (B_c - G)\epsilon$, where $0 < G < B_c$. Otherwise nothing changes and the price stays at its $t = 1$ level. If the above happens however, it goes up further to:

$$P_2^c = \frac{\sigma_F^2}{\sigma_F^2 + (B_c - G)^2 \sigma_\epsilon^2}(F + \epsilon). \tag{16}$$

In this case the behavior of prices is as illustrated in Figure 2.

This simple extension of the model considered previously allows us to make some inferences about the short-run versus long-run behavior of asset prices in case of investor confidence boosted by biased self-attribution. The role of short-sales in producing an asymmetry in prices is again significant. Investigation of the price behavior in the positive news region leads to the discovery of initial positive autocorrelation, followed by a correction caused by the revelation of the asset's value at $t = 3$. As illustrated in Figure 2, the initial $t = 1$ overreaction to a favorable private signal is followed by yet more overreaction at $t = 2$ brought about by another positive news event. Short-sales constraints thus cause prices to exhibit asymmetry

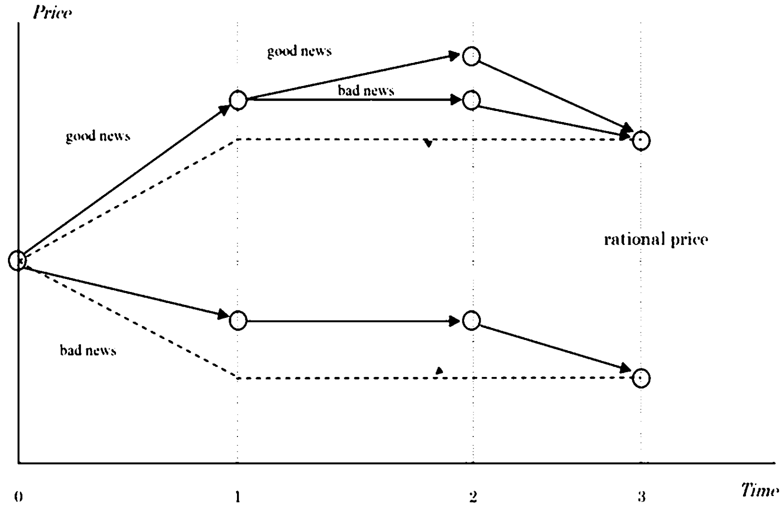


**Figure 2 : Price behavior as a function of time with outcome-dependent investor confidence and short-sales constraints. Confirming public news cause overconfident investors to update their time-2 valuations of the asset's value further upward, resulting in short-run positive return autocorrelation in the good news region.**

in that they initially overreact to good news increasing too much, but then they converge to the true value in a correction phase; on the other hand, prices underreact to bad news and converge to the true value at the final announcement date. Formally, we summarize the above in the following statement.

Proposition 3. *If investor confidence changes because of biased self-attribution, prices after good news exhibit initial short-lag positive autocorrelation ("momentum"):* $\mathrm{Cov}[(\Delta P_2, \Delta P_1)] > 0$, *followed by a correction phase:* $\mathrm{Cov}[(\Delta P_3, \Delta P_2)] < 0$, *and long-lag negative autocorrelation ("reversal"):* $\mathrm{Cov}[(\Delta P_3, \Delta P_1)] < 0$. *In the bad news region, prices display long-lag positive autocorrelation:* $\mathrm{Cov}[(\Delta P_3, \Delta P_1)] > 0$.

## 3 Discussion

Our basic models presented in the previous section may serve as a starting point for the analysis of various financial variables of interest, notably market volatility and trading volume.

Since in the few cases we study above there is only one group of traders present in the market and all of the traders misvalue their (effectively public) signal equally, no trade takes place even though price changes are possible. However, when there are traders with different risky asset valuations present in the market simultaneously, trade does take place and we can analyze the resulting volume.

As the price will also change in the example discussed above, it will be more variable than the underlying fundamentals. Thus our model based on differences of opinion due to biases in confidence can possibly shed light on the excess volatility puzzle.

We have emphasized that it is only a special case when prices in our model can be equal to its counterpart in case when rational traders set it. It is nevertheless instructive to investigate the quality of prices and their role in information revelation. While the investors in our model are not fully rational, they do possess legitimate private information and it is thus meaningful to ask to what extent that information is reflected in market prices. Obviously, in presence of short-sales constraints, one immediately notices that some negative information might remain hidden. The asymmetry in prices resulting from different confidence biases suggests that positive news events will be transmitted with amplification, extreme negative information will be incorporated in prices only gradually.

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