生駒経済論叢　第6巻第2号　2008年10月

Fiscal Policy in a General Equilibrium Model with Imperfect Labor Market

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Abstract　This paper develops a general equilibrium model in which the labor market is imperfect. We investigate the macroeconomic effects of a permanent variation in government expenditure over the long term. We also analyze the optimal provision of public input.

Key words　Government expenditure, Fixed wage, Optimal policy
June 27, 2008 accepted

概要　本論文では，動学的一般均衡モデルにおいて，労働市場に不完全性が存在する場合の生産的公共財の最適供給条件を導出している。固定賃金による労働市場の不完全性のもとでは，生産的公共財の供給は雇用に対して正の影響を与える。したがって，労働市場が不完全な場合，生産的公共財供給は過大となることが示される。

1. Introduction

Numerous papers have extensively analyzed the effects on macroeconomic performance of government expenditures for public infrastructure. A pioneering study presented by Aschauer (1989) estimated the productivity of government expenditure in public infrastructure; it showed that such government expenditures are positively correlated with productivity. Strong and various empirical support exists for the importance of government expenditure in productivity (e.g., Devarajan et al. (1996), Kneller et al. (1999), and Shioji (2001)).

On the theoretical side, several papers describe the effects of government expenditure.(1) Baxter and King (1993), Turnovsky and Fisher (1995), and Chang (1999) introduce government expenditure as an input into the production function in a neoclassical growth model. These studies investigate the macroeconomic effects of a change in government expenditure and show that a fiscal policy consisting of an increase in government expenditure has a positive effect because it directly affects production and indirectly affects both capital and labor productivity. On the other hand, these studies have subsumed a perfect labor market. Furthermore, analysis of government expenditure in an imperfect labor market has not been specifically addressed. This is somewhat surprising because governments in most countries have a strong interest in the issue of attracting investment to boost national employment.

In this paper, we develop a general equilibrium model in which the labor market is imperfect with respect to a fixed wage.(2) We investigate the effects of a permanent variation in government expenditure in the long run. Furthermore, we analyze the optimal provision of public input. Our main contribution is to ad-

(1) Barro (1990) assumes that productive public expenditure positively affects the aggregate production and makes the long-run growth rate an endogenous variable.

(2) Not surprisingly, labor economists have developed various theories of non-competitive wage determination. See reviews of the unemployment literature, with salient studies by Nickell (1990) and Bean (1994) among others. We introduce the fixed wage to our model because authors have indicated possible effects on unemployment of labor market regulations such as minimum-wage provisions (Bazen and Martin (1991)).

dress these issues in a dynamic general equilibrium context with an imperfect labor market.

Results of this paper are summarized as follows. First, we show that, when capital is not substitutable for labor, an increase in government expenditure has a positive effect both on the capital stock and on the output, although an increase in government expenditure might negatively affect both the capital stock and the output when capital is substitutable for labor. This effect is generated by increased marginal productivity of capital and labor.

Second, in contrast to the existing literature, we show that the optimal tax rate on capital is generally non-zero. The logic behind the second result is that increased government expenditure affects the rate of return on capital. However, no household considers it; therefore an externality is generated. The Pigouvian subsidy (tax) is justified when a positive (negative) externality exists.

Finally, we show that the optimally provided public input is undersupplied relative to that in a perfect labor market. The economic intuition is given as the employment effect of fiscal policy in an imperfect labor market. In contrast to the case of a perfect labor market, an increase in government expenditure not only directly affects production; it also indirectly affects production through a positive impact on employment. No indirect effect of fiscal policy on production exists in a perfect labor market. Therefore, the optimally provided public input in an imperfect labor market is undersupplied relative to that in a perfect labor market.

This paper is organized as follows. Section 2 presents the model and solves it. Section 3 presents an investigation into the macroeconomic effects of fiscal policy. Section 4 analyzes the optimal policy under imperfect labor market. Finally, Section 5 concludes this paper.

2. The decentralized economy

Consider a closed, one-good economy in which rational households have the same utility function and firms have the same production technology.

Firms. Final goods y are producible using capital input k, public input g, and

labor input l. The production function is formulated as

$$y=F(k, g, l) \tag{1}$$

The production function F exhibits positive, but diminishing, marginal physical productivity in all factors, $F_i>0(i=k, g, l)$ and $F_{ii}<0(i=k, g, l)$.(3) Furthermore, we assume that $F_{kg}>0$, $F_{lg}>0$, $\Delta \equiv F_{kk}F_{ll}-(F_{kl})^2>0$. Inada conditions are satisfied.

The profit of a representative firm is given as

$$\pi = y-rk-wl=F(k, g, l)-rk-wl \tag{2}$$

where r is the factor price of capital and w is that of labor. The representative firm maximizes its profit, (2), taking r, w, and g as given. The profit maximization conditions are

$$r=F_k(k, g, l), \tag{3}$$

$$w=F_l(k, g, l). \tag{4}$$

Many authors have emphasized the possible effects on unemployment of labor market regulations such as minimum-wage provisions (e.g., Bazen and Martin (1991)). The simple fixed wage model can describe this type of unemployment.(4)

The exogenously fixed wage paid to employed workers is denoted as

$$w=\overline{w} \tag{5}$$

From (4) and (5), the employed labor is given as

$$l=l(k, g), \tag{6}$$

where

$$\frac{\partial l(k, g)}{\partial k} = -\frac{F_{kl}}{F_{ll}} \gtreqless 0 \Leftrightarrow F_{kl} \gtreqless 0, \tag{7}$$

(3) Gramlich (1994) and Glomm and Ravikumar (1997) report evidence about the impact of government expenditure on aggregate output.

(4) This setting is essentially no different from the efficiency wage model presented by Yellen (1984).

$$\frac{\partial l(k,g)}{\partial g} = -\frac{F_{lg}}{F_{ll}} > 0. \tag{8}$$

Households. The representative household is assumed to exist infinitely and to supply one unit of labor inelastically. The population of households is normalized to unity; there is no population growth. Under those circumstances, (6) denotes the employment rate. The lifetime utility function of the household is formulated as

$$U = \int_0^{\infty} u(c)\exp(-\rho t)dt, \tag{9}$$

where c is the consumption, ρ the subjective discount rate, which is a positive constant, $u'(c)>0$ and $u''(c)<0$. The budget constraint is written as

$$\dot{k}=(r-\tau-\delta)k+\bar{w}l+\pi-c-h, \tag{10}$$

where τ is the tax rate on capital, h the head tax, and δ represents the capital depreciation rate.

Each household maximizes lifetime utility (9) subject to a budget constraint (10). The optimality conditions for the household's optimization problem are

$$u'(c)=\lambda, \tag{11}$$

$$\dot{\lambda}=(\rho+\delta+\tau-r)\lambda, \tag{12}$$

where λ is a shadow price of capital. In addition, the transversality condition is

$$\lim_{t\to\infty}\lambda(t)k(t)\exp(-\rho t) = 0$$

From (11), we obtain

$$c=c(\lambda), \tag{13}$$

where $c'(\lambda)=1/u''(c)<0$.

Government. The government provides public input financed by capital and head taxes; its budget constraint can therefore be written as

$$h+\tau k=g. \tag{14}$$

Dynamic system. Next we investigate the existence of the solutions of the dynamic system and stability of the solutions. Using (1)–(6) and (10)–(13), the dynamic system of the macroeconomy is given as

$$\dot{\lambda}=[\rho+\delta+\tau-F_k(k, g, l(k, g))]\lambda, \tag{15}$$

$$\dot{k}=F(k, g, l(k, g))-c(\lambda)-g-\delta k. \tag{16}$$

Solving the above system, we have the following results.

Proposition 1. *There exists a unique stationary equilibrium that exhibits a saddle-point stability in the decentralized economy.*

(Proof) In a stationary equilibrium, $\dot{\lambda}=\dot{k}=0$ holds. Therefore, we have

$$\rho+\delta+\tau=F_k(k, g, l(k, g)), \tag{17}$$

$$c(\lambda)=F(k, g, l(k, g))-g-\delta k. \tag{18}$$

Differentiating the RHS of (17) with respect to k, we obtain

$$\left[F_{kk}+F_{kl}\frac{\partial l}{\partial k}\right]=\left[F_{kk}+\frac{(F_{kl})^2}{F_{ll}}\right]=\frac{F_{kk}F_{ll}-(F_{kl})^2}{F_{ll}}=\frac{\Delta}{F_{ll}}<0.$$

Consequently, the RHS of (17) is decreasing in k. We also have the Inada condition: $\lim_{k\to 0}F_k=+\infty$ and $\lim_{k\to+\infty}F_k=0$. Therefore, a unique k^* exists as a solution of (17). Substituting k in (18) for k^*,

$$c(\lambda^*)=F(k^*, g, l(k^*, g))-g-\delta k^*.$$

Therein λ^* is uniquely determined as well. The linearized system composed of (15) and (16) is

$$\begin{pmatrix}\dot{\lambda}\\ \dot{k}\end{pmatrix}=\begin{pmatrix}J_{11} & J_{12}\\ J_{21} & J_{22}\end{pmatrix}\begin{pmatrix}\lambda-\lambda^*\\ k-k^*\end{pmatrix}, \tag{19}$$

where $J_{11}=0$, $J_{12}=-\Delta\lambda^*/F_{ll}>0$, $J_{21}=-c'(\lambda^*)>0$, and $J_{22}=F_k+F_l\partial l/\partial k-\delta\gtrless 0$. Then, the determinant of the coefficient matrix of (19) is

$$\det J=-J_{12}J_{21}<0.$$

Thus, the stationary equilibrium is a saddle-point. ■

3. Macroeconomic effects of fiscal policy

This section presents an investigation of long-run macroeconomic effects of fiscal policy. We specifically examine a fiscal policy that is financed solely by a head tax ($h>0$ and $\tau=0$) because we wish to examine the long-run effects of changes in government expenditure. For that purpose, it is convenient to first consider the long-run effects of change in government expenditure. Effects of such changes on k^* and y^* are obtained by differentiating the stationary system given in (15) and (16) with respect to g.

The effect of an increased g on k^* can be expressed in the form of

$$\frac{dk^*}{dg}=-\frac{1}{\Delta F_{ll}}\left[F_{kg}-F_{kl}\frac{\partial l}{\partial g}\right]=-\frac{F_{kg}-F_{kl}F_{lg}/F_{ll}}{\Delta F_{ll}}. \qquad (20)$$

Expressions in (20) have been broken down into two sets of effects. The first, given by the first term on a numerator of the RHS of (20), represents the partial effect of change in government expenditure, g, on the marginal productivity of capital. Secondly, the remaining terms represent the partial effect of an increase in g on the marginal productivity of capital through a change in the employment rate. These two effects are positive if $F_{kl}\geq 0$. Then, an increase in g raises the marginal productivity of capital. Thereby, the long-run capital stock, k^*, is increased. However, the second effect is negative if $F_{kl}<0$. An increase in g reduces the marginal productivity of capital when the second effect dominates the first one. An increase in g has negative effect on k^*. These effects (productivity effects) are also present under a perfect labor market (e.g., Baxter and King (1993) and Turnovsky and Fisher (1995)). Therefore, only the productivity effect plays an

important role in determination of long-run capital stock.

Now we examine the effect of an increase in g on y^*. It is expressible in the form:

$$\frac{dy^*}{dg} = F_k\frac{dk^*}{dg} + F_g + F_l\left[\frac{\partial l}{\partial k}\frac{dk^*}{dg} + \frac{\partial l}{\partial g}\right]. \tag{21}$$

A change in government expenditure affects output not only through its change and the effects on capital stock and employed labor. An increase in government expenditure raises long-run capital stock when $F_{kl} \geq 0$. Consequently, the employment rate is also increased and the government expenditure always has a positive output effect. However, an increase in government expenditure might reduce the long-run capital stock when $F_{kl} < 0$. Then, the effect of change in government expenditure is ambiguous. It might negatively affect output.

The above results are summarized as the following proposition.

Proposition 2. *Suppose that the tax rate on capital is zero* ($\tau = 0$). *If* $F_{kl} \geq 0$, *then the fiscal policy has a positive effect on the long-run output,* $dy^*/dg > 0$. *If* $F_{kl} < 0$, *the fiscal policy might negatively affect the long-run output,* $dy^*/dg < 0$.

4. Optimal fiscal policy

In this section, we turn now to the main issue: the derivation of optimal policy. If there is no fixed wage regulation, the first best equilibrium will be attainable to satisfy Kaizuka condition, $F_g = 1$. We shall start by solving the optimization problem that confronts the central planner restricted by fixed wage regulation. Under a fixed wage, the resource constraint of the economy is given as

$$\dot{k} = F(k, g, l(k, g)) - c - g - \delta k. \tag{22}$$

The social planner maximizes the lifetime utility of a representative household, (9), subject to the resource constraint of the economy, (22), restricted by fixed wage regulation. The optimization problem is formulated as

$$\max_{c,g} U = \int_0^{\infty} u(c)\exp(-\rho t)dt,$$

subject to $\dot{k}=F(k, g, l(k, g))-c-g-\delta k.$

Solving the optimization problem, we have the following conditions:

$$u'(c)=\mu, \tag{23}$$

$$\dot{\mu} = \left(\rho+\delta-F_k-F_l\frac{\partial l}{\partial k}\right)\mu, \tag{24}$$

$$F_g+F_l\frac{\partial l}{\partial g} = 1. \tag{25}$$

Therein, μ denotes the social shadow price of capital. The transversality condition is given as

$$\lim_{t\to\infty}\mu(t)k(t)\exp(-\rho t)=0.$$

The optimality condition, (25), implies that the sum of marginal productivity of g and that of labor weighted by $\partial l/\partial g$ (marginal benefit of providing public input) is equal to unity (marginal cost of providing public input). An increase in g always exerts a positive employment effect, $\partial l/\partial g>0$, in an imperfect labor market. The marginal benefit of an increase in g is not only $F_g>0$, but also $F_l\partial l/\partial g$. The cost of an increase in g is equal to unity. Therefore, the optimal condition for providing public input is given as (25) (Modified Kaizuka condition). Then, we have $F_g<1$, in contrast to a perfect labor market.

The decentralized economy is equivalent to the centralized one if $\lambda=\mu$. From (15) and (24), it is necessary and sufficient that the social return of capital is equivalent to the private return of capital:

$$F_k+F_l\frac{\partial l}{\partial k} = F_k-\tau. \tag{26}$$

The optimal tax rate on capital is determined by (26). Indeed, the optimal tax rate is

$$\tau = -F_l \frac{\partial l}{\partial k}. \tag{27}$$

Therefore, the decentralized economy can attain the socially optimal equilibrium when (25) and (27) hold. Thus, under an imperfect labor market, the government uses the tax rate on capital as a policy variable to attain the second best equilibrium, but the government does not use it under a perfect labor market (first best equilibrium).

The above results are summarized as follows.

Proposition 3. *The optimal tax rate on capital and conditions for providing public input are given as*

$$\tau = -F_l \frac{\partial l}{\partial k},$$
$$F_g + F_l \frac{\partial l}{\partial k} = 1,$$

respectively.

The basic argument as to why government chooses a non-zero tax rate on capital can be made as follows: when $F_{kl}>0$, an increase in k increases employment, l, because $\partial l/\partial k>0$. HIowever, agents do not consider an increase in employment in a decentralized economy (positive externality). Therefore, government chooses a negative tax rate on capital. The simplest case is when $F_{kl}=0$ because an increase in k has no power to raise employment (no externality). In that case, the government does not use the capital tax as a policy variable; it sets the tax rate as $\tau=0$. When $F_{kl}<0$, an increase in k decreases the amount of employment, l ($\partial l/\partial k<0$). Agents do not consider $\partial l/\partial k<0$. For that reason, a negative externality exists in the decentralized economy. In that case, the government chooses a positive tax rate to protect jobs.

5. Conclusion

This paper has described a general equilibrium model with productive government expenditure such as public input. The novelty of this model in comparison to those presented in the precedent literature on the macroeconomy and fiscal policy is the incorporation of labor market imperfection. In an imperfect labor market, capital accumulation and productive government expenditure affect labor demand.

Analyzing the model, we have investigated the long-run effects of a fiscal policy consisting of a permanent increase in government expenditure, financed by a lump-sum tax, on capital stock and output. Furthermore, we have derived the optimal policy corresponding to the first-best allocation in the economy with an imperfect labor market. First, we showed that increased government expenditure has no positive long-run effects on capital stock and output for the following reason. Although an increase in government expenditure increases labor demand and marginal productivity of capital, the increase in government expenditure might also decrease the marginal productivity of capital through capital-labor substitution.

Second, we showed that the optimal tax rate on capital is generally non-zero because the source of inefficiency is incorporated into the model as an externality. Although an increase in government expenditure affects the rate of return on capital, households do not consider it. In addition, we showed that public input is undersupplied compared with that in a perfect labor market (first best equilibrium). The logic behind the result is given as the employment effect of fiscal policy in an imperfect labor market. The fact that market allocation is inefficient is crucial for providing optimal policy.

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