Problems of Star Formation

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Abstract

Present status for the study of star formation is reviewed. Especially, notice are
focused on the theory of fragmentation and the role of rotation.

Key Words: Star Formation, Molecular Cloud.

1 Introduction

In recent years, many progress has been achieved in the study of star formation. In the field of ob-
servation, detailed mappings of molecular clouds using weak radio lines emitted by C$^{13}$O, CO$^{18}$,
NH$_3$, etc., are carried out. Also by the develop-ment of the infrared astronomy, IRAS sources
which contain newly born stars are found. Thus, we are now in the position that we have just ob-
tained a detailed knowledge about the structure of a cloud just before the star formation.

In the field of theory, detailed numerical simu-
lations are carried out to see how a collapse and
fragmentation occurs. Thus, we are now in the
position that we have obtained the knowledge
about the hydrodynamics of gravitating media.

So, in this paper, we review a status of the
theory and observation and prepare to construct
a scenario for star formation.

2 Structure of an Observed Molecular Cloud

Stars are formed in molecular clouds. The knowl-
edge about the structure of each molecular cloud is, therefore, essential for the theory of star for-
formation. Recently, detailed structure in a cloud is discriminated by the advance of radio-astronomy.

Observationally, a molecular cloud is made of sheets. Each sheet is made of filaments. Each
filament is made of cores of clouds. These hier-
archical structure is observed typically in Taurus
molecular cloud.

In Fig. 1, we show the position of molecular cores and its radius which are observed by
Heyer(1988). We also show the position of IRAS source found by Myers et al.(1987). In this re-
region, about 40 T Tauri stars exist (Cohen and
Kuhi, 1979).

The mass of each molecular core in this figure
is in the range of $0.3M_\odot$ – $37.6M_\odot$. The radius of
these cores is in the range of 0.1pc – 0.5pc. The
typical density in each core is $10^{-17}$molecules/cm$^3$
($10^{-19.5}$g/cc). The median mass of T Tauri stars
in Taurus cloud, which are newly born stars in these region, is $0.6M_\odot$ (Cohen and Kuhl, 1978), which is consistent with the assumption that the stars are formed from these molecular cores. The average distance between these cores is 0.8pc.

The mass of each filament is about $100M_\odot$. The diameter of the filament is 0.5pc. The length of filament is 5pc.

The separation between filaments is about 10pc in the projected map. Because the sheet of molecular cloud is tilted some 20° with respect to the plane of sky (Kleiner and Dickman, 1984), the mean separation is about 20pc.

Kleiner and Dickman estimated the total mass of the Taurus region using CO$_{13}$ emission power and the optical extinction (Lynds, 1962). The total mass contained in $8° \times 14°$ ($\sim 600pc^2$) region is $5700M_\odot$.

In the Taurus region, the average temperature is about 10K. In the inner part of the cores, we can believe that the temperature is more low. This is because the grain cooling is efficient in this region. The cooling time is $10^7y$. This cooling may trigger the star formation.

#### 3 Structure found by Numerical Simulation

To find how a cloud induces collapse or fragmentation, Miyama et al. (1978) carried out numerical simulations for an isothermal sheet. They gave random velocity fluctuations of the order of 0.1 $c$, and followed the time evolution, where $c$, is the sound velocity. The unperturbed state is an isothermal sheet which extends infinitely to $z$ and $y$ directions and is exerted by external pressure from $z$-direction. In Fig. 2, we show a result. To make it easy to see filamentary structures, we enclose the high density part. In this calculation, they impose periodic boundary condition, so we show the one period by a box. The period in $z$ and $y$ directions is $4\pi H$, where $H$ is given by Eq. (7).

#### 4 Thermal Balance in a Cloud

Since Hattori, Nakano and Hayashi (1969), many calculations are carried out to find the density-temperature relation for an optically transparent interstellar gas. In 1969, CO molecules have not yet been discovered. The most important progress since then is in the inclusion of CO molecules (Kiguchi et al. 1974). We can't find any progress in another important data, i.e., the cosmic ray heating rate, $\zeta$. The surface physics of the grain has been studied steadily mainly by Salpeter’s group.

In the range of density between $10^{-22}g/cc$ and $10^{-18}g/cc$, the gas is heated predominantly by photoelectrons from dust grains and the cosmic ray ionizations of molecular hydrogen. In low density regions, the gas is cooled predominantly by fine structure excitation followed by radiative decay of C$^+$ ions and C atoms, and in high density regions it is cooled by rotational excitation of CO molecules followed by radiative decay. In this region, the time scale of heating and cooling is much shorter than the one for the mechanical equilibrium.

Recent calculation is carried out by de Jong, Dalgarno and Boland (1980). Although the physical process is different, the result is quite similar
with Hattori et al.. The different is in the di-
appearance of a bump in the equation of state,
which was considered to be artificial. The result
of J.D.B is approximated by

$$T = 17K(p/10^{20} \text{g/cc})^{-0.27},$$

i.e., the polytrope index for molecular cloud is
-3.7 (Larson, 1985).

The observational data for the temperature-
density relation are compiled by Myers (1978).
Except the high density end, i.e., $p \sim 10^{-18} \text{g/cc}$,
theory and observation are consistent.

5 Typical Scale of Fragmentation

We can obtain a rough image for the collapse and
the fragmentation of a molecular cloud, using a
dispersion relation. We will summarize here the
result of linear analysis for various cases.

5.1 Infinite uniform sphere

In 1929, Jeans wrote in his famous book about
the gravitational instability of an isothermal gas
cloud using a linear perturbation analysis. He
gave an isothermal perturbation on a uniform in-
finite medium and studied the stability. The time
and spatial dependence of the perturbation has the
form of

$$\delta \rho \propto \exp(i\omega t) \cos(kx), \quad \text{(plane wave)} \tag{2}$$

$$\delta \rho \propto \exp(i\omega t) \frac{1}{r} \sin(kr), \quad \text{(spherical wave)} \tag{3}$$

The results are same for both cases, and are given by

$$\frac{\omega^2}{4\pi G\rho} = \left(\frac{k}{k_j}\right)^2 - 1, \tag{4}$$

where

$$k_j = \sqrt{\frac{4\pi G\rho}{c_s}} \tag{5}$$

and $c_s$ is the isothermal sound velocity.

This expression shows that the most rapidly
growing mode has an infinite wave length, ($k = 0$).
This means that fragmentation is inhibited as a
matter of facts in the spherical overall collapse.
Overall collapse dominates over local fragmenta-
tions.

5.2 Infinite Sheet with Uniform Surface Density

Goldreich and Lynden-Bell (1965) has obtained
the dispersion relation for an infinite sheet with
uniform surface density $\sigma$, neglecting the iner-
tia of the motion in the vertical, i.e., $z$-direction.
They calculated the dispersion relation only for
an incompressible sheet and a polytropic sheet
with index 1. The extension of this calculation
for other polytrope indexes is trivial.

In an isothermal unperturbed equilibrium sheet,
the density distribution in the $z$-direction is
given (Spitzer 1942) by

$$\rho(z) = \frac{\rho_c}{\cosh^2\left(\frac{z}{H}\right)}, \tag{6}$$

where

$$H = \frac{\sigma}{2\rho_c} \tag{7}$$

is an effective half-thickness. For the perturba-
tion of the form:

$$\delta \rho \propto \exp(i\omega t) \cos(k_z z + k_y y) \tag{8}$$

the dispersion relation is approximately given by

$$\frac{\omega^2}{4\pi G\rho} = (k H)^2 - \frac{2k H}{1 + k H}, \tag{10}$$

where

$$k = \sqrt{k_z^2 + k_y^2}$$

is the wave number.

This expression apparently shows that the
most rapidly growing mode has a finite wave
length. The wave length is about $2H$, and the
growth time is about $H/c$. The critical wave-
length above which the sheet is unstable is $H$.

In Fig. 3, we show a dispersion relation for
isothermal sheet and incompressible sheet.

Elmegreen and Elmegreen (1978) studied the
effect of external pressure. If the external pres-
sure is increased from zero to a large value, the
dispersion relation approaches to that for the in-
compressible case.
5.3 Infinite Cylinder with Uniform Line Mass

Ostriker (1964) studied the structure and the stability of a cylinder. In the case of isothermal gas without external pressure, the density distribution is given by

\[ \rho(\tau) = \frac{\rho_c}{(1 + \frac{\tau}{\tau_c})^2}, \quad (12) \]

\[ \sigma_L = \frac{2\kappa^2}{G}, \quad (13) \]

where

\[ R = \sqrt{\frac{\sigma_L}{\pi \rho_c}} \quad (14) \]

is an effective radius of the cylinder.

The Isothermal cylinder has a property similar with the sphere with the polytropes with index 3. When the external pressure \( P_e \) is exerted, the line mass has maximum at \( P_e = 0 \). We show the dispersion relation in Fig. 4 for the perturbation of the form:

\[ \delta \rho \propto \exp(i \omega t + ikz). \quad (15) \]

This shows a sausage type instability. The most rapidly growing mode has a wave length of about \( 2\pi R \), and the growth time is about \( R/c \).

Chandrasekhar and Fermi (1953) has studied the dispersion relation for incompressible cylinder. The result is

\[ \frac{\omega^2}{4\pi G\rho_c} = -2kR I_1(kR)[2K_0(kR) - \frac{1}{I_0(kR)}], \quad (16) \]

where \( R \) is the radius of the cylinder.

The pressure bounded isothermal cylinder approaches the incompressible cylinder when the external pressure increases.

In the limit of long wave length, i.e., \( k \rightarrow 0 \), the dispersion relation for isothermal and incompressible cylinder approaches

\[ \frac{\omega^2}{4\pi G\rho_c} \rightarrow (kr)^2 \log\left(\frac{kR}{2}\right). \quad (17) \]

In the case of sphere,

\[ \frac{\omega^2}{4\pi G\rho_c} \rightarrow -1 \quad (18) \]

and in the case of sheet,

\[ \frac{\omega^2}{4\pi G\rho_c} \rightarrow -kH. \quad (19) \]

Namely, the growth time of large scale perturbation in cylinder is much longer than that in sheet and sphere. The density contrast grows more easily in filament.

5.4 Effect of Rotation

In 1972, Hunter included the effect of rotation. In the dispersion relation, constant additional term
which stabilize the sheet or cylinder appears. For example, the dispersion relation in a thin disk is given by

$$\omega^2 = c^2 k^2 - 2\pi G \sigma k + \kappa^2,$$  \hspace{1cm} (20)

where $\kappa$ is the epicyclic frequency. This is a Coriolis effect.

The rotation velocity can’t be so large due to the stability in the $\tau$-direction. Therefore, rotation has little effect in infinite sheet or cylinder.

5.5 Effect of Magnetic Field

Nakano and Nakamura (1984), Nakano (1988) studied the effect of magnetic fields.

First, consider the case where an uniform magnetic field of strength $B_0$ permeates an isothermal disk perpendicularly. The hydrostatic density distribution in the $z$-direction is, of course, given by Eq. (6). The strength of magnetic field is characterized by the parameter

$$\alpha = \frac{B_0^2}{4\pi^2 G \sigma^2} = \frac{B_0^2}{8\pi \rho c_s^2}. \hspace{1cm} (21)$$

The dispersion relation for an isothermal disk is given by Fig. 5. Compared with the nonmagnetic case, i.e., the case of $\alpha = 0$, which is given by Eq. (10), a disk is stabilized and the most rapidly growing wave length is longer. When $\alpha > 1$, the disk is stabilized completely.

Second, consider the case where an uniform magnetic field is parallel to the disk layer in the unperturbed state. If we assume that the ratio

$$\alpha = \frac{B(z)^2}{c_s^2 \rho(z)} \hspace{1cm} (22)$$

is independent of $z$, the distribution of density and magnetic field is given by

$$\rho(z) = \frac{\rho_c}{\cosh^2(\frac{z}{H})}, \hspace{1cm} (23)$$

$$B(z) = \frac{B_0}{\cosh(\frac{z}{H})}, \hspace{1cm} (24)$$

where

$$H = \frac{c_s \sqrt{1 + \alpha}}{\sqrt{2\pi G \rho_c}}. \hspace{1cm} (25)$$

The dispersion relation is given approximately

$$\frac{\omega^2}{2\pi G \rho_c} = \frac{k^2 H^2}{1 + \alpha} - \frac{2kH}{1 + kH}. \hspace{1cm} (26)$$

In the limit of short wave length ($kH >> 1$), this reduces to the Jeans dispersion relation with the uniform density $\rho$ replaced by the central $\rho_c$.

In the limit of short wave length, the dispersion relation reduces to the Jeans dispersion relation with the uniform density $\rho$ replaced by $\sigma k/2$.

6 Equilibrium and Stability of Rotating Isothermal Cloud

As Heyer (1988) shows, there is a difference of the directions between the axis of rotation and the axis of magnetic polarization. Namely, the effect of rotation dominates over the effect of magnetic field in the cores of molecular clouds. This is yet controversial but the claim is easy to understand.

In the final stage of star formation, i.e., in the collapse and fragmentation of molecular cores, rotation determines the evolution. Therefore, Kiguchi et al. (1987) study the equilibrium configuration and its stability. In Fig. 6, we show a mass-central density ($M - \rho_c$) relation for a isothermal cloud. The angular momentum distribution $j(m/M)$ of these models is given by

$$j(m/M) = J\frac{5}{2}(1 - (1 - m/M)^{2/3}), \hspace{1cm} (27)$$

where $M$ is the total mass, $J$ is the total angular
Further, the axisymmetric instability is studied in detail by Miyama et al. (1989). From these two studies, they obtained the following results:

1. Ranges of the parameters, $\rho_0/\rho_*$ and $\beta$, where a cloud exists stably.
   
   i. Even when the total angular momentum is very large, a cloud with the central density greater that 800$\rho_*$ is unstable to global contraction or expansion.
   
   ii. A cloud with $\beta$ greater than 0.44 is unstable to ring formation.
   
   iii. A cloud with $\beta$ greater than 0.27 is unstable to spiral-arm formation.

   
   i. The maximum mass of a cloud is 4 times the maximum mass of a nonrotating cloud, i.e., even if the total angular momentum is increased, a cloud cannot sustain mass greater than this value because of nonaxisymmetric instability.
   
   ii. The maximum mean rotation velocity is $2c_s$.
   
   iii. The maximum mean density of a cloud is $6\rho_*$.
   
   iv. The maximum height of the boundary surface from the equatorial plane of each cloud is $0.3c_s/\sqrt{GP}$, while the equatorial radius increases as the total angular momentum increases and the maximum flatness of a cloud reaches 4.
   
   3. The internal structure.
      
   i. For an angular momentum distribution corresponding to that in a rigidly-rotating homogeneous sphere, a cloud with $\beta_0$ smaller than 0.6 has a core-envelope structure, where the core is nearly homogeneous and rigidly-rotating while the envelope has the density distribution in proportion to $\varpi^{-2}$. For a cloud with $\beta_0$ greater than 0.6, an outer envelope with a nearly constant density is added and this envelope extends largely with the increase of the total angular momentum. The ring instability begins in this envelope when it becomes very extensive.
      
   ii. When the angular momentum is placed mainly in the outer part such as in the rigidly-rotating case, the ring instability sets in at the equator before the outer envelope develops. When the angular momentum is concentrated in the inner part, the density distribution in the envelope has a steep slope with $\rho \propto \varpi^{-4}$. In this case the ring instability sets in at the center.
      
4. Nonaxisymmetric instability.
   
   i. If $\beta$ is just above the critical value, i.e., $\beta > 0.27$, a two-armed spiral appears as a result of instability. In the case of $\beta \geq 0.34$, a large number of arms appears in the course of evolution.
   
   ii. In the arms, angular momentum is transferred from the inner parts to the outer...
parts through the mechanism of gravitational torque. The time scale of transfer is of the order of rotation period of the arms.

iii. As the final state of the unstable cloud, the inner part is collapsing continuously, while the outer part is extending more and more with time.

5. The quasi-static evolution.

i. The cooling of a cloud may be important for its evolution. It is possible that a cloud initially in equilibrium evolves due to the cooling and collapses to form stars.

7 Summary

In the above, we have reviewed recent studies of fragmentation and equilibrium structure. The coincidence between the observation, simple estimate using the dispersion relation, and the numerical simulation is fine. We have known also the properties of the gravitational equilibrium of a nonspherical cloud. Thus, we have sufficient data for constructing a new scenario of star formation.

References