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Impossibility of collusion under *N* firm yardstick competition[1]

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Abstract　This paper shows that in the yardstick competition the first-best outcome is not vulnerable to collusion among firms.

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1. Introduction

This paper shows that in the yardstick competition (relative performance evaluation) the first-best outcome is not vulnerable to collusion among symmetric firms. Since Shleifer (1985) noted that an important potential limitation of yardstick competition is its susceptibility to collusive manipulation by participating firms, the possibility to prevent collusive conduct has been believed to determine the attractiveness of a particular yardstick scheme. He also pointed out the possibility that collusive strategies may not be sustainable when the number of firms is very large, because firms may simply fail to coordinate on a punishment strategy for one firm that deviates from a collusive equilibrium or they might not know which one of them has violated the collusive agreement. Potters et al. (2004) indicated that in symmetric two-firm repeated games the discriminatory yardstick is much more prone to collusion than the uniform scheme. Our results suggest that the first-best outcome is attainable under the discriminatory scheme for any number of firms.

2. The model

We now describe the model. There are N identical firms. Each firm i ($i = 1,\ldots,N$) acts as a local monopolist facing a downward-sloping demand function $q(p_i)$, which is taken to be identical for all firms. Each has an initial constant marginal cost c_0, and can reduce c_0 to a constant marginal cost c_i by spending $\Psi(c_i)$, where $\Psi(c_0) = 0$, $\Psi'(c_i) < 0$, and $\Psi''(c_i) > 0$. The profits of firm i are given by:

$$V^i = (p_i - c_i)q(p_i) - \Psi(c_i) + T_i, \tag{1}$$

Where T_i is a lump-sum transfer to the firm.

Social welfare in the local market of firm i is defined as the sum of the consumers' surplus and the firm's profit:

$$W^i = \left\{\int_{p_i}^{\infty} q(x)dx\right\} + (p_i - c_i)q(p_i) - \Psi(c_i) \tag{2}$$

In the command optimum, the regulator determines c_i, p_i, and T_i so as to maximize (2) subject to the breakeven constraint $V^i \geq 0$.

Assume that $q(0) < -\Psi'(0)$, $q(c_0) > -\Psi'(c_0)$, and $-q'(c_i) - \Psi''(c_i) < 0$ (the second-order condition for (2)). The social optimum in each market i is then characterized by the first-order conditions:

$$\Psi(c^*) = T^* \tag{3}$$

$$p^* = c^* \tag{4}$$

$$-\Psi'(c^*) = q(p^*) \tag{5}$$

Assume also that the regulator commits himself to the discriminatory price rule and transfer rule given by $p_i = \bar{c}_i$ and $T_i = \bar{\Psi}_i$, and that firms believe this commitment and choose costs accordingly, where

$$\bar{c}_i = \frac{1}{N-1} \sum_{j \neq i} c_j, \tag{6}$$

$$\bar{\Psi}_i = \frac{1}{N-1} \sum_{j \neq i} \Psi(c_j). \tag{7}$$

Then the unique Nash equilibrium cost for each firm i is $c_i = c^*$ and each firm earns a profit of zero, as shown in Shleifer (1985).

3. Results

Before showing the main result, we prove the following lemma.

Lemma 1. All firms choose the same costs and earn zero profits under collusion.

Proof. Since $\sum_{i=1}^{N} \sum_{j \neq i} \Psi(c_j)/(N-1) = \sum_{i=1}^{N} \Psi(c_i)$, the sum of the profits of the firms is given by:

$$\sum_{i=1}^{N} V^i = \sum_{i>j}\left(\frac{c_i-c_j}{N-1}\right)\left[q\left(\frac{c_i+C_{-i,j}}{N-1}\right)-q\left(\frac{c_j+C_{-i,j}}{N-1}\right)\right], \tag{8}$$

Where $C_{-i,j}=\sum_{k\neq i,j} c_k$. This must be non-positive, as the demand curve is downward sloping. Thus, this is maximized when all firms choose the same costs, and attains the value of zero. **Q.E.D.**

Since the unique Nash equilibrium maximizes joint profits, and all firms earn zero profits at this point, the unique subgame-perfect equilibrium of an infinitely repeated version of the game comprises repetition of this one-shot equilibrium. This result is stated as follows.

Proposition 1. **(Collusion in a Long-term Relationship)**
Collusion is sustainable in the long run only when all firms repeatedly choose the Nash equilibrium cost c^*.

Proof. Since the profit of firm i is $(\bar{c}_i-c_i)q(\bar{c}_i)-\Psi(c_i)+\bar{\Psi}_i$, it can ensure zero profits for itself by selecting c^c when the other firms are choosing c^c. So if its best response is some $c_i\neq c^c$, then it must make positive profits. (From the assumptions, we have $q(c^c)<-\Psi'(c^c)$ when $c^c<c^*$ and $q(c^c)>-\Psi'(c^c)$ when $c^c>c^*$. Thus, it can increase its profits by unilaterally inflating (deflating) its cost when $c^c<c^*$ ($c^c>c^*$), respectively.) From expression (8), at least one firm must make losses.

We suppose that each firm j has adopted the following trigger strategy:(2)

Choose c^c as long as the firm earns zero profits. If it has made losses in the previous period or, if it has not chosen c^c in the previous period, then choose c^* forever after.

From Lemma 1, the present value of profits that firm receives from choosing c^c repeatedly is at most $V^i(c^c,c^c)/(1-\delta)=0$, where $\delta\in[0,1)$ is the discount factor of the firm. On the other hand, the present value of profits that firm i receives by deviating optimally from c^c is at least $V^i(c_i^*(c^c),c^c)+\delta V^i(c^*,c^*)/(1-\delta)=$

(2) For the trigger strategy, see Gibbons (1992).

$V^i(c_i^*(c^c), c^c) > 0$. Hence, the best deviation from the collusive action c^c is profitable for every firm i. Since there is no cost c_i such that $V^i(c_i, c^*) > 0$ for the Nash equilibrium cost c^*, collusion is a subgame-perfect equilibrium outcome only if $c^c = c^*$. **Q.E.D.**

4. Comparison of results for collusion

In this section, we discuss the relation between our results on collusion and those of experimental study by Potters et al. (2004). To do so, we consider the two-firm model with linear demand $q(p) = \alpha - \beta p$ $(\alpha, \beta > 0)$ and quadratic R & D costs $\Psi(c) = \gamma(c - c_0)^2$ $(\gamma > 0)$. We assume that $\alpha/2\gamma < c_0 < \alpha/\beta$ and $\beta < 2\gamma$, so that all the assumptions mentioned before are satisfied.

In this case, the total profit of firm i is written as:

$$\begin{aligned} V^i &= (c_j - c_i)q(c_j) - \Psi(c_i) + \Psi(c_j). \\ &= [c_j - c_i][\gamma c_i + (\gamma - \beta)c_j - (2\gamma c_0 - \alpha)] \end{aligned} \tag{9}$$

Thus, the unique Nash equilibrium is $(c_1, c_2) = \left({}^{2\gamma c_0 - \alpha}\!/_{2\gamma - \beta}, {}^{2\gamma c_0 - \alpha}\!/_{2\gamma - \beta}\right)$.
Define $V^i_{++,1} \equiv \left\{(c_1, c_2) \middle| c_j < c_i < -{}^{\gamma - \beta}\!/_{\gamma}\, c_j + {}^{2\gamma c_0 - \alpha}\!/_{\gamma}\right\}$.
and $V^i_{++,2} \equiv \left\{(c_1, c_2) \middle| -{}^{\gamma - \beta}\!/_{\gamma}\, c_j + {}^{2\gamma c_0 - \alpha}\!/_{\gamma} < c_i < c_j\right\}$. Then the set of marginal costs that attain a positive level of profit for firm i is defined as $V^i_{++} \equiv V^i_{++,1} \cup V^i_{++,2}$. We write V^i_{+} when the strict inequalities are replaced by weak inequalities, and write the boundary of the set V^i_{+} as V^i_0. It can be verified from (9) that $V^1_0 \cap V^2_0 = \left({}^{2\gamma c_0 - \alpha}\!/_{2\gamma - \beta}, {}^{2\gamma c_0 - \alpha}\!/_{2\gamma - \beta}\right)$, which equals to the symmetric Nash equilibrium point.

Since $0 < \beta < 2\gamma$, we have ${}^{\gamma - \beta}\!/_{\gamma} < 1 < {}^{\gamma}\!/_{\gamma - \beta}$ if $0 < \beta < \gamma$, and ${}^{\beta - \gamma}\!/_{\gamma} < 1 < {}^{\gamma}\!/_{\beta - \gamma}$ if $\gamma < \beta < 2\gamma$. In both cases, $V^i_{++,1} \cap V^j_{++,2} = \phi$. By the definitions, $V^i_{++,1} \cap V^j_{++,1} = \phi$, $V^i_{++,2} \cap V^j_{++,2} = \phi$, so that $V^i_{++} \cap V^j_{++} = \phi$. Obviously, $V^i_{++} \cap V^j_0 = \phi$ holds.

We can prove a related result.

Proposition 2. The Nash equilibrium outcome (c^*, c^*) is a minimax point of the profit function of firm $i (i = 1, 2)$, so that it is a solution to the problem:

$$\min_{c_j} \max_{c_i} V^i(c_i, c_j). \tag{10}$$

We therefore have $\min_{c_j} \max_{c_i} V^i(c_i, c_j) = V^i(c^*, c^*) = 0$.

Proof. From similar arguments to Proposition 1, if the best response of firm i is some $c_i \neq c_j$, then it must make positive profits. Thus, from (8), if firm j chooses cost c_j such that the best response of firm i is some $c_i \neq c_j$, then firm i makes positive profits, and firm j earns losses. If firm j were to deviate to $c_j = c_i$ it would eliminate its own losses as well as profits of firm i. Hence the symmetric Nash equilibrium and the minimax point must coincide. **Q.E.D.**

Experimental results obtained by Potters et al. (2004) indicated that the discriminatory yardstick competition is prone to collusion in repeated game settings. However, the payoff structure of their game is different from here, because they omit the transfer rule $T_i = \Psi(c_j)$. Their parameter values are $\alpha = 34$, $\beta = 0.5$, $\gamma = 1$, $c_0 = 25$, and $c^c = 20$. (The present assumptions are satisfied for $17 < c_0 < 68$ and $\beta < 2\gamma$.) Under these assumptions we obtain $-\Psi(c_i)+\Psi(c_j) = 50c_i - c_i^2 - 50c_j + c_j^2$, the best response of firm i, $c_i^*(c_j) = 8 + c_j/4$, and the relation $V^i(c_i^*(c^c), c^c) > V^i(c^c, c^c) = V^i(c^*, c^*)$, where $V^i(c_i^*(c^c), c^c) = 49$ and $V^i(c^c, c^c) = V^i(c^*, c^*) = 0$. The relation among payoffs implies that collusion is not sustainable in the long run. This is because even the trigger strategy using the maximum punishment c^* cannot deter deviations from collusive action c^c.

Potters et al. (2004) used a benefit of slack function $B(c_i) = 40c_i - c_i^2$, and obtained $c_i^*(c_j) = 3 + c_j/4$ and the inequality $V^i(c_i^*(c^c), c^c) > V^i(c^c, c^c) > V^i(c^*, c^*)$, where $c_i^*(c^c) = 8$ and $V^i(c_i^*(c^c), c^c) = 544$, and $V^i(c^c, c^c) = 400$, and $c^* = 4$ and $V^i(c^*, c^*) = 144$. The payoff structure of their model is thus the same as that of the Cournot duopoly.(3) This is why collusion is sustainable in their model.

(3) For collusion in the Cournot duopoly, see Gibbons (1992).

References

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